Math115 2011 Test 4

March 23$^{\text{rd}}$, 2011

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

1. We are given the following five data points; (-3,-5), (-1,1), (0,1), (1,-4), (3,4)

   (a) Find the best fit straight line for the points. [4]

   (b) Set up the five equations for a best fit cubic equation $y = ax^3 + bx^2 + cx + d$ to the points and calculate $(-3)^n + (-1)^n + 0^n + 1^n + 3^n$ for $n = 0, \ldots, 6$, explaining why half of these sums are zero. [2]

   (c) Use these sums to solve the matrix equation using row operations and hence find the best fit cubic to the points. Which data points are closest to the curve? [6]

2. Suppose we have two quantities which are related as follows:

$$a_{n+1} := \left(\frac{1}{3}\right) a_n + \left(\frac{-1}{3}\right) b_n$$

$$b_{n+1} := \left(\frac{5}{2}\right) a_n + \left(\frac{-3}{2}\right) b_n$$

$$a_0 := 22 \quad , \quad b_0 := 10$$

   (a) Find the values of a_1, a_2, b_1 and b_2 using the formulae. [2]

   (b) Given that the eigenvectors of the underlying matrix are $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ diagonalise the matrix and hence find the general formula for a_k and b_k. [5]

   (c) For which n does one of a_n or b_n first drop below 1 in absolute value? [1]