Math115 2011 Test 5

March 30^{th}, 2011

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

1. We are given these two planes:

$$P: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + k \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \quad , \quad Q: \begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 1 \\ -5 \\ 3 \end{pmatrix} = 1$$

   (a) What is the equation of P in dot product form? Find a vector equation for Q. [5]

   (b) Use the dot product form of both planes to find the line where they intersect. [3]

   (c) Show that $R: \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ is a point on P and find the point where the line from R perpendicular to P intersects Q. Give an example of a line through R which never intersects with Q. [4]

2. Set up a 3×3 matrix equation and hence find the shortest distance between these two lines: [8]

$$F: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 1 \end{pmatrix} + r \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix} \quad , \quad G: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$$