University College of Cape Breton MATH115

Introduction to Matrix Algebra

June 2003 Time : 3 hours

Clearly write your answers to the questions showing all working and checks and indicate what each mathematical calculation is doing. The best SIX answers will be counted.

Q1. One of these sets is a vector space. Prove or disprove each of the three vector space rules for these three sets, indicating which set is the vector space. [10]

$$\{(x,y) \mid y \geq |x|\}, \quad \{(x,y,z) \mid y = 3x - 2z\}, \quad \{(x,y,z) \mid z = (y-1)(x-1) - 1\}$$

Q2. (a) What is the distance from the point $P := (-3,1,2)$ to the point $Q := (-1,3,1)$? [2]

(b) Find the shortest distance from P to the plane $L := \{x + 3z = 4y\}$. [3]

(c) What is the equation of the plane parallel to L which passes through Q? [1]

(d) Find equations of two lines in L which are orthogonal. [4]

Q3. Using these three matrices A B and C, calculate the following matrices, or explain why they are not defined: A^2, A^{-1}, B^{-1}, C^2, $(BC)^{-1}$, $(AB - B)$, $(CA)^T$, $\det(B)$. [10]

$$A := \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} \quad B := \begin{bmatrix} 1 & -5 & -3 \\ 1 & -3 & 0 \end{bmatrix} \quad C := \begin{bmatrix} 4 & 1 \\ 2 & -1 \\ -3 & 3 \end{bmatrix}$$

Q4. (a) Find the other eigenvalues of this matrix given that one of them is -3. [4]

$$\begin{bmatrix} 7 & -10 & -5 \\ 10 & -18 & -10 \\ -10 & 20 & 12 \end{bmatrix}$$

(b) Find all three eigenvectors. [6]

Q5. We are given the matrix $M := \begin{bmatrix} 1 & -3 & 2 & -5 \\ 1 & -5 & 3 & -7 \\ 1 & -1 & 1 & -3 \\ 1 & 1 & 0 & -1 \\ 0 & -2 & 1 & -2 \end{bmatrix}$

(a) Use row operations to reduce the augmented matrix to reduced row-echelon form and thus find the general solution to $M\underline{x} = \begin{bmatrix} -13 \\ -20 \\ -6 \\ 1 \\ -7 \end{bmatrix}$ [5]

(b) Explain why the rank of M is 2 from the first part of the question, and give a basis for the columnspace of M, and express all of the columns in terms of it. [5]

Q6. (a) Prove that this set of vectors is independent. [3]

$$V := \left\{ \begin{pmatrix} 1 \\ -2 \\ -1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 3 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \\ 3 \\ -1 \\ -2 \end{pmatrix} \right\}$$

(b) Use the Gram Schmidt process to form an orthonormal basis for V. [7]

Q7. (a) Use row and column operations and Laplace expansions to evaluate the determinant of this matrix: [4]

$$\begin{bmatrix} 1 & -2 & -a & -2 \\ -1 & b & 3 & 0 \\ -2 & 1 & -2 & -2 \\ -1 & 2 & 3 & 2 \end{bmatrix}$$

(b) For which values of a or b is this matrix non-invertible? [1]

(c) Substitute $a = 1$ and $b = 1$ into the matrix and find the nullspace. [5]

END OF QUESTION PAPER