Cape Breton University

Math 1204

Matrix Algebra

February 2012

Time : $\frac{3}{2}$ hours

Please answer any THREE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. (a) Use row operations to find the inverse of $C := \begin{pmatrix} -3 & 3 & 4 \\ -4 & 5 & 2 \\ 4 & 3 & 5 \end{pmatrix}$, and do not introduce fractions until you have pivoted twice. [9]

(b) Calculate the determinant of $\begin{pmatrix} -3 & x & 4 \\ -4 & 5 & 2 \\ y & 3 & 5 \end{pmatrix}$. Check whether the values for x and y from C give a number which appears as the denominator in C^{-1}. [3]

Q2. (a) Diagonalise this matrix and hence give the formula for M^k for any k. [8]

$$M := \frac{1}{3}\begin{pmatrix} 5 & 14 \\ 4 & 4 \end{pmatrix}$$

(b) Check your answer for $k = 0$, $k = -1$, $k = 1$ and $k = 2$. [4]

Q3. (a) Solve this equation in matrix algebra for X explaining what you are doing at each simplification step: [4]

$$3(BA + AX) = A + C^T$$

(b) What sizes must the matrices be if A is $m \times n$? Which matrices need to have inverses for X to have a unique solution? How does wanting a unqiue solution affect the sizes of each matrix? [4]

(c) Using the following matrices, substitute them into your solution for (a) and hence find X (which should also be all integers). [4]

$$A := \begin{pmatrix} 11 & 9 \\ 4 & 3 \end{pmatrix}, \quad B := \begin{pmatrix} 3 & -12 \\ -4 & 16 \end{pmatrix}, \quad C := \begin{pmatrix} 1 & 26 \\ 15 & 54 \end{pmatrix},$$

Q4. We will be dealing with this matrix in this question:

$$H := \begin{pmatrix} -17 & 22 & -28 \\ 88 & -104 & 142 \\ 76 & -91 & 123 \end{pmatrix}$$

(a) Check that $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ is an eigenvector of H and identify its eigenvalue. [2]

(b) Evaluate the determinant of $(H - \lambda I)$ by means of a co-factor expansion and factorise it (the polynomial should have very small coefficients). [5]

(c) Find one other eigenvector of H (if you couldn't get another eigenvalue from (b) ask me for one). [5]

END OF QUESTION PAPER