Cape Breton University

Math 1204

MATRIX ALGEBRA

April 2013

Time : 3 hours

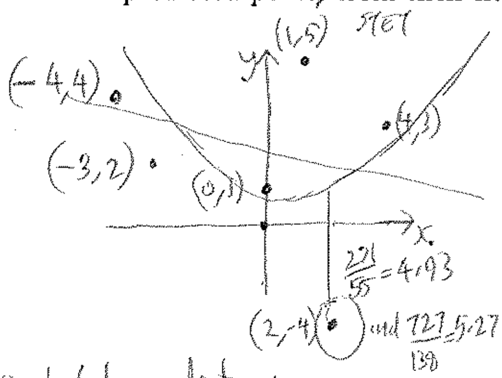
Please answer any FIVE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. We have the following six data points; (-4,4), (-3,2), (0,1), (1,5), (2,-4), (4,3)

(a) Find the best fit quadratic equation through this data, using adjoint method [8]

(b) Re-use your working to identify the equation of the best fit straight line. [2]

(c) Sketch the points and both best fit equations on the same graph and predict which data points are the furthest from each. Find the exact distance of ~~the~~ your predicted points from their fitted values [2]



$\frac{221}{55}=4.93$ and $\frac{727}{138}=5.27$

$$A=\begin{pmatrix}16&-4&1\\9&-3&1\\0&0&1\\1&1&1\\4&2&1\\16&4&1\end{pmatrix}\qquad A^TA=\begin{pmatrix}610&-18&46\\-18&46&0\\46&0&6\end{pmatrix}$$

$$A^Ty=\begin{pmatrix}16&9&0&1&4&16\\-4&-3&0&1&2&4\\1&1&1&1&1&1\end{pmatrix}\begin{pmatrix}4\\2\\1\\5\\-4\\3\end{pmatrix}=\begin{pmatrix}64+18+5-16+48\\-16-6+5-8+12\\4+2+1+5-4+3\end{pmatrix}=\begin{pmatrix}119\\-13\\11\end{pmatrix}$$

Best fit quadratic:

$$\left(\begin{array}{ccc|c}610&-18&46&119\\-18&46&0&-13\\46&0&6&11\end{array}\right)$$

$R_1 \Leftarrow R_1 - 8R_3$

$$\left(\begin{array}{ccc|c}242&-18&-2&31\\-18&46&0&-13\\46&0&6&11\end{array}\right)$$

$R_3 \Leftarrow R_3 + 3R_1$

$$\left(\begin{array}{ccc|c}242&-18&-2&31\\-18&46&0&-13\\772&-54&0&104\end{array}\right)$$

$R_3 \Leftarrow R_3 + R_2$

$$\begin{array}{cccc}242&-18&-2&31\\-18&46&0&-13\\754&-8&0&91\end{array}$$

$$Adj(A^TA)=\begin{pmatrix}276&+108&-2116\\+108&1544&-828\\-2116&-828&27736\end{pmatrix}^T$$

$610\times276-18\times108-46\times2116$
$=\cancel{267640}\ 69080$

$$=\frac{1}{69080}\begin{pmatrix}276&+108&-2116\\+108&1544&-828\\-2116&-828&27736\end{pmatrix}\begin{pmatrix}119\\-13\\11\end{pmatrix}=\frac{1}{69080}\times\begin{pmatrix}8164\\-16328\\ \end{pmatrix}=\frac{1}{110}\begin{pmatrix}13\\-26\\102\end{pmatrix}$$

$$y=\frac{13x^2-26x+102}{110}=\frac{13(x-1)^2+89}{110}$$

(b) $\begin{pmatrix}46&0\\0&6\end{pmatrix}\begin{pmatrix}m\\b\end{pmatrix}=\begin{pmatrix}-13\\11\end{pmatrix}$ $\quad m=\frac{-13}{46}$ $\quad b=\frac{11}{6}$

Q2. (a) Given that one eigenvector of the matrix $B := \begin{pmatrix} -109 & -67 & -78 \\ 144 & 90 & 102 \\ 28 & 16 & 21 \end{pmatrix}$ is $\begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$,

find one eigenvalue and then the other two eigenvalues by polynomial division. [~~4~~] 6

(b) Find an eigenvector of B that does not belong to a dominant eigenvalue by using row operations. [~~6~~] 4

(c) Explain ~~three~~ several sorts of different behaviour we could expect ~~from~~ of $B^k v_0$ as k increases with respect to the~~se~~ ~~particular~~ eigenvalues of B and different choices for v_0. [2]

$$B\underline{v} = \begin{pmatrix} -109+268-156 \\ 144-360+204 \\ 28-64+42 \end{pmatrix} = \begin{pmatrix} 3 \\ -12 \\ 6 \end{pmatrix} = 3\begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} \quad \text{so } \lambda_1 = 3$$

$$\det(B-\lambda I) = (-109-\lambda)\big((90-\lambda)(21-\lambda)-16\times 102\big) + 67\times\big(144\times(21-\lambda)-28\times 102\big)$$
$$-78\times\big(144\times 16-28\times(90-\lambda)\big)$$
$$= (-109-\lambda)(\lambda^2-111\lambda+258)+67\times(-144\lambda-168)$$
$$-78\times(-216+28\lambda)$$
$$= -\lambda^3+2\lambda^2+12099\lambda-258\lambda-28122$$
$$-9648\lambda \cancel{-172056} + 11256 + 16848 - 2184\lambda$$
$$= -\lambda^3+2\lambda^2+9\lambda-18$$

$$\begin{array}{r|l} & \lambda^2+\lambda-6 \\ \hline -\lambda+3 & -\lambda^3+2\lambda^2+9\lambda-18 \\ & -\lambda^3+3\lambda^2 \\ & \quad -\lambda^2+9\lambda \\ & \quad -\lambda^2+3\lambda \\ & \qquad 6\lambda-18 \\ & \qquad 6\lambda-18 \\ & \qquad\quad 0 \end{array}$$

$\lambda^2+\lambda-6 = (\lambda+3)(\lambda-2)$

$\lambda = 2$ is not dom

$$\left(\begin{array}{ccc|c} -111 & -67 & -78 & 0 \\ 144 & 88 & 102 & 0 \\ 28 & 16 & 19 & 0 \end{array}\right)$$

$R_1 \Leftarrow -R_1$, $R_2 \Leftarrow R_2 - 5R_3$
$$\left(\begin{array}{ccc|c} 111 & 67 & 78 & 0 \\ 4 & 8 & 7 & 0 \\ 28 & 16 & 19 & 0 \end{array}\right)$$

$R_3 \Leftarrow R_3 - 2R_2$, $R_1 \Leftarrow R_1 - 8R_2$
$$\left(\begin{array}{ccc|c} 79 & 3 & 22 & 0 \\ 4 & 8 & 7 & 0 \\ 20 & 0 & 5 & 0 \end{array}\right)$$

$R_3 \Leftarrow R_3 \times \frac{1}{5}$
$$\left(\begin{array}{ccc|c} 79 & 3 & 22 & 0 \\ 4 & 8 & 7 & 0 \\ 4 & 0 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -9 & 3 & 0 & 0 \\ -24 & 8 & 0 & 0 \\ 4 & 0 & 1 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} -3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 \end{array}\right) \quad \boxed{v = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}}$$

$R_1 \Leftarrow R_1 \times \frac{1}{3}$ $R_2 \Leftarrow R_2 - \frac{8}{3}$ old R_1

$$B^k \underline{v}_0 = \sim 3^k + \sim(-3)^k + \sim 2^k$$

Q3. (a) Which vector space axioms are false or true for the following sets of points in the (x, y) plane? [4]

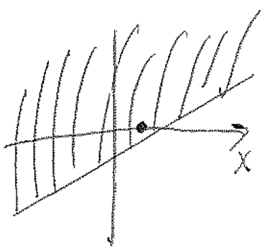
$y > |x|$

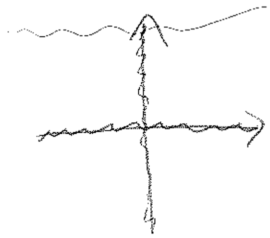
$i)\ y \geq x-1 \quad ii)\ xy = 0 \quad iii)\ \cancel{y^2 \leq 3}$ ~~(choose 2 of them)~~

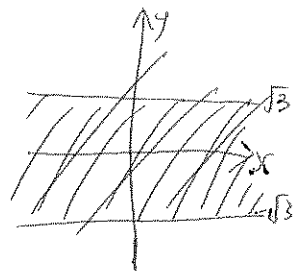
(b) ~~Find~~ create two lines which are wholly within in the plane $P := \begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = 11$

but which do not ever intersect. [3] 2

(c) Find the equation of a line $\perp$ within P which is perpendicular to your lines from (b) ~~using the Gram-Schmidt method~~ and determine where your line intersects with them both. [3] 4







$(1,0)$ isM but $0 \geq 1-1=0$

$(1,0)+(1,0)=(2,0)$ $0 \not\geq 1$

and $2\times(1,0) =$

$(0,0)$ isM as $0 \geq -1$

$(1,0)$ and $(0,1)$ are M $0\times 1=0$

$(0,0)$ is M $0\times 0=0$

But $(1,0)+(0,1)$ is not $1\times 1=1$

$\alpha(x,y)=(\alpha x,\alpha y)$

if $xy=0$ then $(\alpha x)(\alpha y) = \alpha^2 xy = \alpha^2 0 = 0$

$(0,0)$ not M $0 \not> 0$

$(0,1)$ is M $1>|0|$

but $(0,-1)$ is not $-1 \not> |0|$

$(x_1,y_1)+(x_2,y_2)=(x_1+x_2, y_1+y_2)$

$y_1+y_2 > |x_1|+|x_2| \geq |x_1+x_2|$ ✓

(b) $(5\ -2\ 3 : 0)$

$5x+3z=2y$

eg $y=4$ $x=z=1$ is a direction

$\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ is a point $5+0+6=11$

also $\begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$ is a point $0+2+9=11$

so $L_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} t$

$L_2 = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} s$

(c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} t + \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} k$

$\begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} - \frac{22}{18}\begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 18-11 \\ 45-44 \\ 0-11 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ -11 \end{pmatrix}$ ✓

$\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ 1 \\ -11 \end{pmatrix} k = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} s$

$\begin{pmatrix} -7 & 1 \\ -1 & 4 \\ +11 & 1 \end{pmatrix}\begin{pmatrix} k \\ s \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\left(\begin{array}{cc|c} -7 & 1 & 1 \\ 27 & 0 & -3 \\ 18 & 0 & -2 \end{array}\right)$ $k = -\frac{1}{9}$ $\frac{1}{9}\begin{pmatrix} 9-7 \\ 0-1 \\ 18+11 \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 2 \\ -1 \\ 29 \end{pmatrix}$

Q4. (a) Evaluate the determinant of $M := \begin{pmatrix} 4 & 2 & 3 \\ 3 & -7 & x \\ y & 2 & 3 \end{pmatrix}$ and explain why there is no value of x or y which guarantees that M is non-singular. [4]

(b) Determine the the ~~adjoint~~ inverse of M in terms of x and y.

$$\text{(a)}\ \det(M) = 4\times(-21-2x) - 2\times(9-xy) + 3\times(6+7y)$$
$$= -84 - 8x - \cancel{18} + 2xy + \cancel{18} + 21y$$
$$= 2xy - 8x + 21y - 84$$
$$= 2x(y-4) + 21(y-4)$$
$$= (2x+21)(y-4)$$

So $\det(M) = 0$ when

$(2x+21)(y-4) = 0$

ie $y = 4$ or $x = \frac{-21}{2}$

whatever x is we can choose $y = 4$.

$$\mathrm{Adj}(M) = \begin{pmatrix} -21-2x & -(9-xy) & 6+7y \\ -(6-6) & 12-3y & -(8-2y) \\ 2x+21 & -(4x-9) & -28-6 \end{pmatrix}^T = \begin{pmatrix} -21-2x & 0 & 2x+21 \\ xy-9 & 12-3y & 9-4x \\ 7y+6 & 2y-8 & -34 \end{pmatrix}$$

$$\text{so}\quad M^{-1} = \frac{1}{(2x+21)(y-4)} \begin{pmatrix} -(2x+21) & 0 & 2x+21 \\ xy-9 & 3(4-y) & 9-4x \\ 7y+6 & 2(y-4) & -34 \end{pmatrix}$$

$$M^{-1}M = \frac{1}{(2x+21)(y-4)} \begin{pmatrix} (-4+y)(2x+21) & (2x+21)(-2+2) & (2x+21)(-3+3) \\ 4xy-36+36-9y+9y+4xy & & \\ & & \end{pmatrix}$$

Q5. In this question we will be studying the recurrence $c_{j+1} := 7c_j \overset{+0c_{j-1}}{-} 36c_{j-2}$ where $c_0 = 41$, $c_1 = -211$ and $c_2 = 203$.

(a) Substitute the given values to get c_3 and c_4. [2]

(b) Find the eigenvalues underlying this recurrence and then use the diagonalisation method to get the ~~solution~~. formula for c_n for any n. [8]

(c) Use logarithms to predict for which value of j we will first have $c_j > 70000000$ and check whether it is accurate. [2]

(a) $c_3 = 7\times 203 - 36\times 41 = -55$ $\qquad c_4 = 7\times -55 - 36 \times -211 = 7211$

(b) $-\lambda^3 + 7\lambda^2 - 36 = 0$

$\lambda = -2 \qquad 8 + 28 - 36 = 0$

$$\begin{array}{r|l} & \lambda^2 - 9\lambda + 18 \\ \hline -\lambda + 2 & -\lambda^3 + 7\lambda^2 + 0\lambda - 36 \\ & -\lambda^3 + 2\lambda^2 \\ \hline & 9\lambda^2 + 0\lambda \\ & 9\lambda^2 + 18\lambda \\ \hline & -18\lambda - 36 \\ & -18\lambda - 36 \\ \hline & 0 \end{array}$$

$\lambda^2 - 9\lambda + 18 = (\lambda - 6)(\lambda - 3) = \lambda^2 - 6\lambda - 3\lambda + 18$

so $\lambda_1 = -2 \quad \lambda_2 = 3 \quad \lambda_3 = 6$

$$\text{so} \begin{pmatrix} c_{n+2} \\ c_{n+1} \\ c_{n} \end{pmatrix} = \begin{pmatrix} 4 & 9 & 36 \\ -2 & 3 & 6 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} (-2)^n & 0 & 0 \\ 0 & 3^n & 0 \\ 0 & 0 & 6^n \end{pmatrix} \begin{pmatrix} 4 & 9 & 36 \\ -2 & 3 & 6 \\ 1 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 203 \\ -211 \\ 41 \end{pmatrix}$$

$$= \begin{pmatrix} \sim\sim\sim \\ \sim\sim\sim \\ (-2)^n \;\; 3^n \;\; 6^n \end{pmatrix} \begin{pmatrix} 71 \\ -37 \\ 7 \end{pmatrix}$$

solving $\left(\begin{array}{ccc|c} 4 & 9 & 36 & 203 \\ -2 & 3 & 6 & -211 \\ 1 & 1 & 1 & 41 \end{array}\right)$

$\left(\begin{array}{ccc|c} -5 & 0 & 27 & -166 \\ -5 & 0 & 3 & -334 \\ 1 & 1 & 1 & 41 \end{array}\right)$

$R_1 \Leftarrow R_1 - R_2$

$\left(\begin{array}{ccc|c} 0 & 0 & 24 & 168 \\ -5 & 0 & 3 & -334 \\ 1 & 1 & 1 & 41 \end{array}\right)$

$R_1 \Leftarrow R_1 \times \frac{1}{24}$

$R_2 \Leftarrow R_2 - 3R_1$

$R_3 \Leftarrow R_3 - R_1$

$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 7 \\ -5 & 0 & 0 & -355 \\ 1 & 1 & 0 & 34 \end{array}\right)$

$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 7 \\ 1 & 0 & 0 & 71 \\ 0 & 1 & 0 & -37 \end{array}\right)$

$c_n = 71\times(-2)^n - 37\times 3^n + 7\times 6^n$

(c) so $c_n \approx 7\times 6^n$

and $7\times 6^n > 70000000$

$n > \frac{\log(10000000)}{\log 6} = 8.996$

so try $n=9$ $\quad c_n = -36352 - 728271 + 70543872$

$= 69779249$ so need c_{10}

Q6. (a) Simplify this equation $E(XA + 3I) + 2A^TA = 0$ to find X in simplest terms of the other matrices, assuming inverses exist where needed, giving reasons [4]

(b) Find the inverses of both of these matrices and substitute the matrices into your solution for (a) to see if you get integers for X. [3]

$$A := \begin{pmatrix} 4 & 5 \\ -1 & -2 \end{pmatrix}, \quad E := \begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix}$$

(c) If A was a 1×2 matrix what sizes must X and E be for (a) to possibly hold? Why? ~~Must an~~ Find an A and X E exist for all possible choices of A and X of these sizes? [5]

(a)
$$E(XA + 3I) = -2A^TA$$
$$XA + 3I = -2E^{-1}A^TA$$
$$XA = -2E^{-1}A^TA - 3I$$
$$X = -2E^{-1}A^T - 3A^{-1}$$

(b)
$$A^{-1} = \frac{1}{-8+5}\begin{pmatrix} -2 & -5 \\ 1 & 4 \end{pmatrix} = \frac{+1}{3}\begin{pmatrix} 2 & 5 \\ -1 & -4 \end{pmatrix}$$

$$E^{-1} = \frac{1}{20-18}\begin{pmatrix} 4 & -6 \\ -3 & 5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 4 & -6 \\ -3 & 5 \end{pmatrix}$$

so
$$X = -\cancel{2} \times \frac{1}{\cancel{2}}\begin{pmatrix} 4 & -6 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 5 & -2 \end{pmatrix} - \frac{3}{3}\begin{pmatrix} 2 & 5 \\ -1 & -4 \end{pmatrix}$$

$$= -\left(\begin{pmatrix} 16-30 & 12-4 \\ 25-12 & 3-10 \end{pmatrix} + \begin{pmatrix} 2 & 5 \\ -1 & -4 \end{pmatrix}\right) = -\begin{pmatrix} -12 & 13 \\ 12 & -11 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & -13 \\ -12 & 11 \end{pmatrix}$$

(c) A is 1×2 so A^T is 2×1 and A^TA is 2×2

XA implies X is n×1 so XA is n×2 but I only exists for square

so n=2 and X is 2×1

E is therefore 2×n = 2×2

we need XA+3I to be non invertible so eg
$$\begin{pmatrix} x \\ 1 \end{pmatrix}\begin{pmatrix} a & 1 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} xa & x \\ a & 1 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} xa+3 & x \\ a & 4 \end{pmatrix}$$

so $4(xa+3) - xa = 0$
$$3xa = -12$$
$$xa = -4$$
$$x = 4 \quad a = -1$$

$p+q = 2 \quad p+q = \frac{1}{2}$

so
$$E\begin{pmatrix} -1 & 4 \\ -1 & 4 \end{pmatrix} = -2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$$

$$\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} -1 & 4 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} -p-q & 4p+4q \end{pmatrix} = \begin{pmatrix} -2 & 2 \end{pmatrix}$$

Q7. (a) Show that these vectors are not independent in $\mathbb{R}^4$ and find a dependence equation. [4] [5]

$$v_1 := \begin{pmatrix} 1 \\ -1 \\ -2 \\ 1 \end{pmatrix}, \quad v_2 := \begin{pmatrix} -4 \\ 3 \\ 1 \\ 4 \end{pmatrix}, \quad v_3 := \begin{pmatrix} -2 \\ -1 \\ -3 \\ +2 \end{pmatrix}, \quad v_4 := \begin{pmatrix} 3 \\ 2 \\ 1 \\ 3 \end{pmatrix}$$

(b) Find the unique vector which is perpendicular to all of these vectors and hence find the dot product equation of the hyperplane they span. [8] [5]

(c) ~~Explain why~~ Verify the distance^s^ from v_2 ~~and~~ to v_4 v_3 to v_4 ~~is~~ are the same and determine whether v_1 is closer to v_3 or v_4. [3] [2]

$$\left(\begin{array}{cccc|c} 1 & -4 & 2 & 3 & 0 \\ -1 & 3 & 1 & 2 & 0 \\ -2 & 1 & 3 & 1 & 0 \\ 1 & 4 & -2 & 3 & 0 \end{array}\right)$$

$a_1 = 3a_2$
$a_2 = a_2$ so
$a_3 = 2a_2$
$a_4 = -a_2$

$3\underline{v}_1 + \underline{v}_2 + 2\underline{v}_3 - \underline{v}_4 = \underline{0}$

$\underline{v}_4 = 3\underline{v}_1 + \cancel{4}\underline{v}_2 + 2\underline{v}_3$ ✓

$$\left(\begin{array}{cccc|c} 1 & -4 & 2 & 3 & 0 \\ 0 & -1 & 3 & 5 & 0 \\ 0 & -7 & 7 & 7 & 0 \\ 0 & 8 & -4 & 0 & 0 \end{array}\right)$$

$R_3 \leftarrow R_3 \times \frac{1}{7}$ $\quad R_4 \leftarrow R_4 \times -\frac{1}{4}$

$$\left(\begin{array}{cccc|c} 1 & -4 & 2 & 3 & 0 \\ 0 & -1 & 3 & 5 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 0 \\ 0 & 5 & 0 & 5 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & +2 & -1 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & -2 & +1 & 0 & 0 \end{array}\right)$$

(b)

$$\left(\begin{array}{cccc|c} 1 & -1 & -2 & 1 & 0 \\ -4 & 3 & 1 & 4 & 0 \\ 2 & 1 & 3 & -2 & 0 \\ 3 & 2 & 1 & 3 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -1 & -2 & 1 & 0 \\ -1 & 0 & -5 & 7 & 0 \\ 3 & 0 & 1 & -1 & 0 \\ 5 & 0 & -3 & 5 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 7 & -1 & 0 & -1 & 0 \\ 14 & 0 & 0 & 2 & 0 \\ 3 & 0 & 1 & -1 & 0 \\ 14 & 0 & 0 & 2 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 14 & -1 & 0 & 0 & 0 \\ 7 & 0 & 0 & 1 & 0 \\ 10 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\underline{n} = \begin{pmatrix} 1 \\ 14 \\ -10 \\ -7 \end{pmatrix}$ so $\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 14 \\ -10 \\ -7 \end{pmatrix} = 0$

(c) $\underline{v}_4 - \underline{v}_2 = \begin{pmatrix} 7 \\ -1 \\ 0 \\ -1 \end{pmatrix}$ $\quad 7^2 + 1^2 + 0^2 + 1^2 = 51$

$\underline{v}_4 - \underline{v}_3 = \begin{pmatrix} 5 \\ 3 \\ 4 \\ 1 \end{pmatrix}$ $\quad 5^2 + 3^2 + 4^2 + 1^2 = 51$

$\underline{v}_4 - \underline{v}_1 = \begin{pmatrix} 2 \\ 3 \\ 3 \\ 2 \end{pmatrix}$ $\quad 26$

$\underline{v}_1 - \underline{v}_3 = \begin{pmatrix} 3 \\ 0 \\ 1 \\ -1 \end{pmatrix}$ $\quad 11$ $\quad \underline{v}_3$ closer

END OF QUESTION PAPER