Cape Breton University

Math 1204

MATRIX ALGEBRA

April 2015

Time : 3 hours

Please answer any FIVE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. Suppose that X satisfies this equation: $(2B^{-1} + XA)^{-1} = \frac{1}{3}A^{-1}B$ and A and B have inverses

(a) Show that if X is ~~the~~ all zeros then A must be a multiple of the identity matrix. and B is invertible [4]

(b) ~~Now assuming that both A and B have inverses,~~ solve the equation for X, giving all rules used for simplification. [5]

(c) Letting $X = B$ now explain why we must have $A = I$ and that $B^2 = I$. Give an example of a B which is not the identity matrix which satisfies this equation [3]

(a) $(2B^{-1})^{-1} = \frac{1}{2}B = \frac{1}{3}A^{-1}B$ so $\frac{1}{6}(3I - 2A^{-1})B = 0$ $A^{-1} = \frac{3}{2}I$ $A = \frac{2}{3}I$

(b) $2B^{-1} + XA = \left(\frac{1}{3}A^{-1}B\right)^{-1} = 3B^{-1}(A^{-1})^{-1} = 3B^{-1}A$

$XA = 3B^{-1}A - 2B^{-1} = B^{-1}(3A - 2I)$

$X = B^{-1}(3A - 2I)A^{-1} = 3B^{-1} - 2B^{-1}A^{-1}$

(c) If $X = B$ then $B^2 = (3A - 2I)A^{-1} = (3I - 2I)I^{-1} = II^{-1} = I$

$B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ $B^2 = \begin{pmatrix} (-1)^2 & 0 \\ 0 & (-1)^2 \end{pmatrix} = I$

$2B^{-1} + XA = 3B^{-1}A$

$XA = 3B^{-1}A - 2B^{-1}$

$= B^{-1}(3A - 2I)$

$X = B^{-1}(3A - 2I)A^{-1}$

Q2. Consider the recurrence $c_{k+1} = 2c_k + \cancel{3}^{9} c_{k-1} - \cancel{6}^{18} c_{k-2}$ and $c_0 := 45$, $c_1 := 50$, $c_2 := 190$.

(a) Find c_3 from the recurrence and find the eigenvalues of the underlying matrix by factoring a cubic ~~equation~~ polynomial. Give the eigenvectors in standard form. [~~4~~ 3]

(b) Form P, the matrix of eigenvectors, and use the adjoint method to find its inverse and hence give the formula for c_k in terms of powers of its eigenvalues. [6]

(c) Explain what the behaviour of c_k will be as k goes to infinity, why the dominant eigenvalue situation is unusual this time and a set of initial values (that are not simple powers) which would keep increasing indefinitely. [2]

increasing

(a) $c_3 = 2c_2 + 9c_1 - 18c_0 = 380 + 450 - \cancel{270}\,810 = \cancel{260}\ 20$

$x^3 - 2x^2 - 9x + 18 = (x^2 - 9)(x - 2) = (x-3)(x+3)(x-2)$

so $\lambda_1 = 3 \quad \lambda_2 = -3 \quad \lambda_3 = 2$ and $\underline{v}_1 = \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix}$ $\underline{v}_2 = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$ $\underline{v}_3 = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$

(b) $\mathrm{Adj}\left(\begin{pmatrix} 9 & 9 & 4 \\ 3 & -3 & 2 \\ 1 & 1 & 1 \end{pmatrix}\right) = \begin{pmatrix} -5 & -(3-2) & 3+3 \\ -(9-4) & 9-4 & -(9-9) \\ 18+12 & -(18-12) & -27-27 \end{pmatrix}^T = \begin{pmatrix} -5 & -5 & 30 \\ -1 & 5 & -6 \\ 6 & 0 & -54 \end{pmatrix}$

$P \times \mathrm{Adj}(P) = \begin{pmatrix} -45-9+24 & -45+45 & 270-54-216 \\ & & \\ & & \end{pmatrix} = \begin{pmatrix} -30 & 0 & 0 \\ 0 & -30 & 0 \\ 0 & 0 & -30 \end{pmatrix}$

(c) $\begin{pmatrix} c_{k+2} \\ c_{k+1} \\ c_k \end{pmatrix} = \begin{pmatrix} 9 & 9 & 4 \\ 3 & -3 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3^k & 0 & 0 \\ 0 & (-3)^k & 0 \\ 0 & 0 & 2^k \end{pmatrix} \frac{-1}{30} \begin{pmatrix} -5 & -5 & 30 \\ -1 & 5 & -6 \\ 6 & 0 & -54 \end{pmatrix} \begin{pmatrix} 190 \\ 50 \\ 45 \end{pmatrix}$

$= \begin{pmatrix} \ldots \\ \ldots \\ 3^k \ (-3)^k \ 2^k \end{pmatrix} \begin{pmatrix} -5 \\ 7 \\ 43 \end{pmatrix}$ so $c_k = -5 \times 3^k + 7 \times (-3)^k + 43 \times 2^k$

$\to 2 \times 3^k + 43 \times 2^k$ k even

$-12 \times 3^k + 43 \times 2^k$ k odd

$3^k + 2^k = 2, 5, 13, \ldots$

Q3. **(a)** Use row operations to find the dependency equation between these four vectors and check it holds. [8]

$$\underline{v}_1 := \begin{pmatrix} 3 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \underline{v}_2 := \begin{pmatrix} 3 \\ 2 \\ -1 \\ -1 \end{pmatrix}, \quad \underline{v}_3 := \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix}, \quad \underline{v}_4 := \begin{pmatrix} 1 \\ 2 \\ 0 \\ 3 \end{pmatrix}$$

(b) Show that $\underline{n} := \begin{pmatrix} -1 \\ 2 \\ 2 \\ -1 \end{pmatrix}$ is perpendicular to v_1, v_2, v_3 and v_4. If $L := \begin{pmatrix} 5+4\alpha \\ 5-2\alpha \\ -3+4\alpha \\ -6+\alpha \end{pmatrix}$ is a line, use $\underline{n}$ to find the point where L intersects the vector space containing the vectors from part **(a)**. [4]

(a)
$$\left(\begin{array}{cccc|c} 3 & 3 & 2 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 2 & 3 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 0 & 6 & -1 & 1 & 0 \\ 0 & 3 & 0 & 2 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 0 & 6 & 0 & 4 & 0 \\ 0 & 3 & 0 & 2 & 0 \\ 1 & -1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 3/2 & 0 & 1 & 0 \\ 1 & 7/2 & 0 & 0 & 0 \\ 0 & -9/2 & 1 & 0 & 0 \end{array}\right) \leftarrow \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 3/2 & 0 & 1 & 0 \\ 1 & -1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right)$$

$\alpha_2 = k$
$\alpha_4 = -\frac{3}{2}k$
$\alpha_1 = -\frac{7}{2}k$
$\alpha_3 = \frac{9}{2}k$

so $$-7\underline{v}_1 + 2\underline{v}_2 + 9\underline{v}_3 - 3\underline{v}_4 = \begin{pmatrix} -21+6+18-3 \\ -7+4+9-6 \\ -7-2+9+0 \\ -7-2+18-9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \checkmark$$

(b) $\underline{u} \cdot \underline{v}_1 = -3+2+2-1 = 0$
$\underline{u} \cdot \underline{v}_2 = -3+4-2+1 = 0$ ✓
$\underline{u} \cdot \underline{v}_3 = -2+2+2-2 = 0$
$\underline{u} \cdot \underline{v}_4 = -1+4+0-3 = 0$

$$\begin{pmatrix} 5+4\alpha \\ 5-2\alpha \\ -3+4\alpha \\ -0+\alpha \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \\ -1 \end{pmatrix} = 0 = -5-4\alpha+10-4\alpha-6+8\alpha+6-\alpha$$
$= 5-\alpha$
so $\alpha = 5$

and point is $\begin{pmatrix} 25 \\ -5 \\ 17 \\ -1 \end{pmatrix}$

Q4. (a) If $M := \begin{pmatrix} -14 & -6 & 9 \\ -72 & -20 & 36 \\ -90 & -30 & 49 \end{pmatrix}$, find and factor the determinant of $M - \lambda I$. [6]

(b) Find two eigenvectors of the repeated eigenvalue ~~and then~~ manipulate them to make an orthogonal basis set for these eigenvectors, checking $M\underline{v} = \lambda \underline{v}$. [6]

a) $\det\begin{pmatrix} -14-\lambda & -6 & 9 \\ -72 & -20-\lambda & 36 \\ -90 & -30 & 49-\lambda \end{pmatrix} \overset{R_2 \to R_2 - 4R_1}{=} \det\begin{pmatrix} -14-\lambda & -6 & 9 \\ -16+4\lambda & 4-\lambda & 0 \\ -90 & -30 & 49-\lambda \end{pmatrix}$

$C_1 \to C_1 + 4C_2$

$= \det\begin{pmatrix} -38-\lambda & -6 & 9 \\ 0 & 4-\lambda & 0 \\ -210 & -30 & 49-\lambda \end{pmatrix}$

$= (4-\lambda) \times (\lambda^2 - 49\lambda + 38\lambda - 38 \times 49 + 9 \times 210)$

$= (4-\lambda)(\lambda^2 - 11\lambda + 28) = (4-\lambda)(\lambda-7)(\lambda-4)$

$= -\lambda^3 + [illegible]\lambda^2 - 72\lambda + 112$

so $\lambda_1 = \lambda_2 = 4 \quad \lambda_3 = 7$

(b) $M - 4I = \begin{pmatrix} -18 & -6 & 9 \\ -72 & -24 & 36 \\ -90 & -30 & 45 \end{pmatrix} \xrightarrow{\text{so}} \left(\begin{array}{ccc|c} -6 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$3z = 2y + 6x \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ 2/3 \end{pmatrix} y$

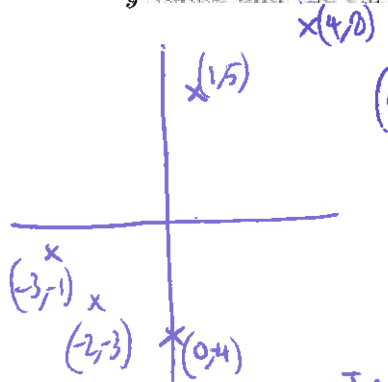
so $\underline{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad \underline{u}_2 = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} \qquad \underline{e}_2: \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} - \frac{0+0+4}{1+0+5}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} - \frac{2}{3}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$M \times \begin{pmatrix} -2 \\ 9 \\ 2 \end{pmatrix} = \begin{pmatrix} 28-54+18 \\ 144-180+72 \\ 180-270+98 \end{pmatrix} = \begin{pmatrix} -8 \\ 36 \\ 8 \end{pmatrix} = 4 \times \begin{pmatrix} -2 \\ 9 \\ 2 \end{pmatrix} \qquad = \begin{pmatrix} -2/3 \\ 3 \\ 2/3 \end{pmatrix} = \begin{pmatrix} -2 \\ 9 \\ 2 \end{pmatrix} \times \frac{1}{3}$

Q5. Data is collected and the following points are found (0,-4), (1,5), (-2,-3), (-3,-1), (4,8).

(a) Find the best fit quadratic by solving the appropriate 3×3 matrix equation. [8]

(b) Plot the points, the quadratic and calculate the fractional differences between the y values and the curve values. [4]



(a) $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & -2 & 1 \\ 9 & -3 & 1 \\ 16 & 4 & 1 \end{pmatrix}$ $\quad \underline{w} = \begin{pmatrix} -4 \\ 5 \\ -3 \\ -1 \\ 8 \end{pmatrix}$

$A^T A = \begin{pmatrix} 354 & 30 & 30 \\ 30 & 30 & 0 \\ 30 & 0 & 5 \end{pmatrix}$ $\quad A^T \underline{w} = \begin{pmatrix} -4+5-3-1+8 \\ 0+5+6+3+32 \\ 0+5-12-9+128 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ \cancel{46} \\ 112 \end{pmatrix} \begin{pmatrix} 112 \\ 46 \\ 5 \end{pmatrix}$

$64-8-27+1$
$=56-26=30$

$1+16+81+256$
$=98+256=354$

$\begin{pmatrix} 354 & 30 & 30 & \vdots & 112 \\ 30 & 30 & 0 & \vdots & 46 \\ 30 & 0 & 5 & \vdots & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 324 & 0 & 30 & \vdots & 66 \\ 30 & 30 & 0 & \vdots & 46 \\ 30 & 0 & 5 & \vdots & 5 \end{pmatrix}$

$\downarrow$

$y = \frac{1}{4}x^2 + \frac{77}{60}x - \frac{1}{2}$ $\quad \begin{pmatrix} 1 & 0 & 0 & \vdots & 1/4 \\ 0 & 30 & 0 & \vdots & 77/2 \\ 0 & 0 & 5 & \vdots & -\frac{10}{4} \end{pmatrix} \leftarrow \begin{pmatrix} 144 & 0 & 0 & \vdots & 36 \\ 30 & 30 & 0 & \vdots & 46 \\ 30 & 0 & 5 & \vdots & 5 \end{pmatrix}$

$= \frac{1}{60}(15x^2 + 77x - 30)$

(b) $x=0 \quad y = \frac{-30}{60} \qquad x=-2 \quad y = \frac{124}{60} \qquad x=4 \quad y = \frac{548}{60}$

$x=1 \quad y = \frac{62}{60} \qquad x=-3 \quad y = \frac{126}{60}$

Q6(a)

A Rank 1 $\begin{pmatrix}4&-4&4\\-4&4&-4\\4&-4&4\end{pmatrix}$ $R_3 \leftarrow R_3 - R_1$, $R_2 \leftarrow R_2 + R_1$ $\begin{pmatrix}4&-4&4\\0&0&0\\0&0&0\end{pmatrix}$

B Rank 2 $\begin{pmatrix}4&4&4\\4&-4&4\\-4&4&-4\end{pmatrix}$ $R_3 \leftarrow R_3 + R_2$, $R_1 \leftarrow R_1 + R_2$ $\begin{pmatrix}8&0&8\\4&-4&4\\0&0&0\end{pmatrix}$ $R_2 \leftarrow R_2 - \frac{1}{2}R_1$ $\begin{pmatrix}8&0&8\\0&-4&0\\0&0&0\end{pmatrix}$

C Rank 3 $\begin{pmatrix}4&4&4\\4&4&-4\\4&-4&-4\end{pmatrix}$ $R_1 \leftarrow R_1 + R_3$, $R_2 \leftarrow R_2 + R_3$ $\begin{pmatrix}8&0&0\\8&0&-8\\4&-4&-4\end{pmatrix} \to \begin{pmatrix}8&0&0\\0&0&-8\\0&-4&4\end{pmatrix} \to \begin{pmatrix}8&0&0\\0&0&-8\\0&-4&-4\end{pmatrix}$

$BA = \begin{pmatrix}16&-16&16\\48&-48&48\\-48&48&-48\end{pmatrix}$ Rank = 1 $CA = \begin{pmatrix}16&-16&16\\-16&16&-16\\16&-16&16\end{pmatrix}$ Rank = 1

Since $A\underline{v}=0$ for two $\underline{v}$s $(BA)\underline{v} = B(A\underline{v}) = B0 = 0$ and $CA=0$ too
so two homog sol^ns

(b) $\det(A) = \det(B) = 0$ expanding by row 3

$\det(C) = 8 \times \det\left(\begin{pmatrix}0&-8\\-4&0\end{pmatrix}\right) = 8 \times (0-32) = 256$

For a 2x2 with only 4s we can have $\begin{pmatrix}4&-4\\4&4\end{pmatrix}$ with $\det = 16+16 = 32$

so theoretical max would be $4\times32 - 4\times-32 + 4\times32 = 128\times3 = 384$
and min $M = -384$

However if we try to achieve that

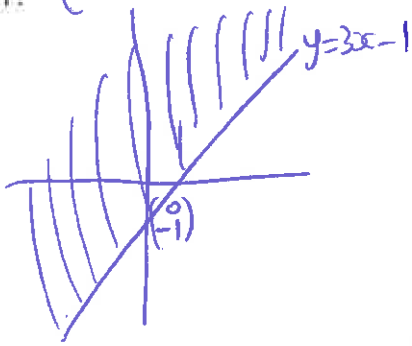
$\begin{pmatrix}-4&4&x\\4&4&-4\\4&4&4\end{pmatrix}$ gives $-4\times32 - 4\times32 + x\times0$
$= -256$

$\begin{pmatrix}4&-4&-4\\-4&4&-4\\-4&-4&4\end{pmatrix} = 0 + 4\times-32 - 4\times32$

with only 4s a 2x2 has either 32, 0 or -32,
so min non zero is 32, but that can't be achieved

$\begin{pmatrix}4&4&y\\-4&4&-4\\-4&4&4\end{pmatrix}$ $128 - -128 + 0 = 256$

Q7. (a)



$\underline{0}$ is in $0 > 3\times 0 - 1 = -1$ ✓

$\begin{pmatrix}-1\\-3\end{pmatrix}$ is in $-3 > -3-1=-4$ ✓

$\begin{pmatrix}2\\6\end{pmatrix}$ is in $6 > 3\times 2 - 1 = 5$ ✓

But

$\begin{pmatrix}-1\\-3\end{pmatrix} + \begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix}$ which is in

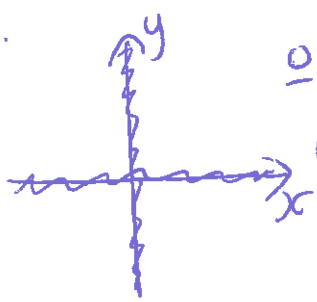
$y_1 > 3x_1 - 1$ $\quad y_1+y_2 > 3x_1+3x_2-2$

$y_2 > 3x_2 - 1$ $\quad = 3(x_1+x_2)-1+1$

$x=\frac{2}{3}$ $y=\frac{11}{10}$ $\frac{11}{10} > \frac{3}{2}-1=\frac{1}{2}$

But $\alpha = 2$

$x=\frac{4}{3}$ $y=\frac{22}{10}$ $\frac{22}{10} > 4-1=3$ No.



$\underline{0}$ $x=0$ $y=0$

$xy = 0\times 0 = 0$

$\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$

$1\times 0=0$ $\quad 0\times 1=0$ $\quad 1\times 1 = 1 \neq 0$

$\alpha\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\alpha x\\ \alpha y\end{pmatrix}$ and $(\alpha x)(\alpha y) = \alpha^2 xy = 0$

(b) $\underline{n}\cdot\begin{pmatrix}3\\2\\-4\end{pmatrix}=0$ $\quad \underline{n}\cdot\begin{pmatrix}5\\3\\-7\end{pmatrix}=0$

$\left(\begin{array}{ccc|c}3&2&-4&0\\5&3&-7&0\end{array}\right) \to \left(\begin{array}{ccc|c}3&2&-4&0\\2&1&-3&0\end{array}\right)$

$\downarrow$

$\left(\begin{array}{ccc|c}-1&0&2&0\\0&1&1&0\end{array}\right) \leftarrow \left(\begin{array}{ccc|c}-1&0&2&0\\2&1&-3&0\end{array}\right)$

so $\underline{n} = \begin{pmatrix}2\\-1\\1\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}3\\-5\\2\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\1\end{pmatrix} = 6+5+2$
$= 13$

$\left(\begin{array}{ccc|c}2&-1&1&13\\-3&4&2&12\end{array}\right)$

$\left(\begin{array}{ccc|c}2&-1&1&13\\-7&6&0&-14\end{array}\right)$

$\left(\begin{array}{ccc|c}2&-1&1&13\\1&-6/7&0&2\end{array}\right)$

$\left(\begin{array}{ccc|c}0&5/7&1&9\\1&-6/7&0&2\end{array}\right)$

so $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\0\\9\end{pmatrix} + \begin{pmatrix}6\\7\\-5\end{pmatrix}\frac{y}{7}$

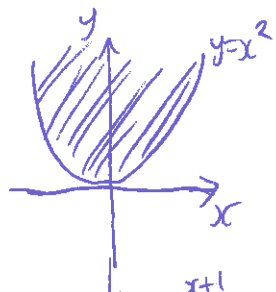
a line

END OF QUESTION PAPER

Q6. (a) Which of the vector space axioms are true for these relations? Draw the areas within the regions describe and give counterexamples or explain why the axioms are true. [6]

$$y \ge x^2 \quad -1 < |y - x| < 1$$

(b) Find the dot product form of the plane P which is defined by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + \beta \begin{pmatrix} 5 \\ 3 \\ -7 \end{pmatrix}$ and where it intersects with $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} = 12$ [6]



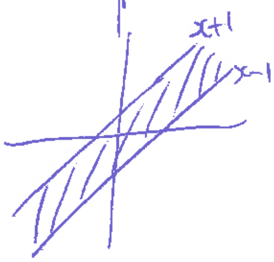
$0 \ge 0^2 = 0$ ✓

$\left.\begin{matrix} y_1 \ge x_1^2 \\ y_2 \ge x_2^2 \end{matrix}\right\} y_1 + y_2 \ge x_1^2 + x_2^2 = (x_1+x_2)^2 - 2x_1x_2 \ge (x_1+x_2)^2$ unless $x_1 < 0 \;\; x_2 > 0$

but $(x_1+x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$

$(1,1)$ is in but $(2,2)$ is not

$1 \ge 1^2$ $\qquad 2 \not\ge 2^2 = 4$



(b) $\underline{n}$: $\left(\begin{array}{ccc|c} 3 & 2 & -4 & 0 \\ 5 & 3 & -7 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 3 & 2 & -4 & 0 \\ 2 & 1 & -3 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} -1 & 0 & +2 & 0 \\ 2 & 1 & -3 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} -1 & 0 & +2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right)$

$\underline{n} = \begin{pmatrix} +2 \\ -1 \\ 1 \end{pmatrix}$ ✓

so $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} \circ \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \cancel{-78 - 105 + 2 = -281}$ $6 + 5 + 2 = 13$

$\left(\begin{array}{ccc|c} 2 & -1 & 1 & 13 \\ -3 & 4 & 2 & 12 \end{array}\right) \to \left(\begin{array}{ccc|c} 2 & -1 & 1 & 13 \\ -7 & 6 & 0 & -14 \end{array}\right) \to \left(\begin{array}{ccc|c} 2 & -1 & 1 & 13 \\ 1 & -6/7 & 0 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & 5/7 & 1 & 9 \\ 1 & -6/7 & 0 & 2 \end{array}\right)$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6/7 \\ 7/7 \\ -5/7 \end{pmatrix} y + \begin{pmatrix} 2 \\ 0 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 9 \end{pmatrix} + \begin{pmatrix} 6 \\ 7 \\ -5 \end{pmatrix} t$