Cape Breton University

Math1204

MATRIX ALGEBRA

February 2015 | Time : 2 hours

Answer any THREE of these questions, giving all of your working and reasoning.

Q1. (a) Use row operations to find the homogeneous and particular solutions for: [7]

$$2\,w - 3\,x + y + z = 0 \quad , \qquad -7\,w + 8\,x + 4\,y + 3\,z = -1$$

$$w - 4\,x + 8\,y + 7\,z = -1 \quad , \qquad -3\,w + 2\,x + 6\,y + 5\,z = -1$$

(b) Verify that $w = 3$, $x = 2$, $y = 4$, $z = -4$ is indeed a solution by substituting these values into (a) and then find it as a combination of your solutions. Find an all-integer solution with all of w, x, y and z non-negative and less than 5. [4]

Q2. (a) Use the adjoint method (showing all details of all steps) on $A := \begin{pmatrix} 4 & 2 & 3 \\ t & -4 & -3 \\ -1 & 2 & 3 \end{pmatrix}$.

Multiply A by your answer and deduce the determinant of A. [7]

(b) Explain why A can never be singular for any value of t. Create a 3×3 matrix with two unknowns in which has a constant determinant but explain why almost every such matrix must be singular. [4]

Q3. (a) Suppose we have the following recurrence;

$$b_{n+1} := b_n + 20 \times b_{n-1} \;, \quad b_0 = 19 \;, \quad b_1 = -58$$

Find b_2 then use diagonalisation to get b_k for any positive integer k. [7]

(b) Use logarithms to find which value of k makes b_k negative for the last time. [4]

Q4. Use a determinant column operation and a row operation to factorise $\det(E - \lambda I)$ and hence find all eigenvectors of $E := \begin{pmatrix} 21 & -8 & 16 \\ 16 & -3 & 16 \\ -16 & 8 & -11 \end{pmatrix}$. [11]

END OF QUESTION PAPER