Math1204 Test 4

March 10^{th} 2015

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The questions are weighted as shown and can be answered in any order.

1. The twin recurrences in this question are

$$\begin{aligned} a_{n+1} &= -3a_n + \frac{9}{5}b_n \\ b_{n+1} &= -\frac{12}{5}a_n + \frac{6}{5}b_n \end{aligned}$$

   (a) Diagonalise the underlying matrix and hence find the formula for a_k and b_k in general if $a_0 = 900$ and $b_0 = 1300$. [8]

   (b) Using your answer for (a), determine for which values of k are a_k and b_k positive, explaining why. [4]

2. (a) Find the best fit straight line to this data using the matrix method. [5]

x_j	3	4	6	7	10
y_j	-3	1	1	2	4

   (b) Identify which data point lies exactly on the best fit line and which is furthest away vertically from it. Verify that the sum of the vertical differences between the y_j and the best fit line is 0 by adding fractions (not decimals). [3]