Math1204 Assignment 5: late March 2015

Answer all questions and give complete reasons and *checks* for your answers, writing words to explain your conclusions. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The questions are weighted as shown. Because of everyone having different numbers, do not expect nice round numbers to arise. Feel free to use Maple to assist you in verifying your calculations, but do not use decimals to approximate.

Hand in your rough working as well so I can see how you investigated and reached your final results. You are reminded that plagiarism is a serious offense and when it is detected you will be punished. You are allowed to talk with me or other students in general about the questions but the work you hand in should be your own.

The numbers represented by a, b, c and d should be replaced by the largest four digits of your registration number in non-increasing order. For instance, if my registration number was 20105023 then i would take $a = 5$, $b = 3$, $c = 2$ and $d = 2$.

Let P be the plane $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} c \\ d \\ a \end{pmatrix} = 11$ and let L be the line $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} + \alpha \begin{pmatrix} -2 \\ 7 \\ -5 \end{pmatrix}$.

1. (a) Derive a vector equation for P with two direction vectors and use Gram-Schmidt to get an orthogonal basis for the two direction vectors. Find two points in $\mathbb{R}^3$ with all integer values that lie on P. [4]

   (b) Verify that your given normal has zero dot product with your direction vectors and your basis vectors and use Gram-Schmidt with your basis vectors and a randomly chosen vector to verify that you do indeed get a multiple of the normal vector out. [3]

   (c) Find the point of intersection of P and L by combining the two equations. Give a theoretically possible registration number (that nobody else in the class chooses, email me to reserve yours) that would make P and L parallel. [4]

2. (a) Given these vectors in $\mathbb{R}^4$, show they are not independent, find a dependency equation and a spanning set and hence determine the dimension of the spanning space. [7]

$$v_1 := \begin{pmatrix} 4 \\ -5 \\ 4 \\ 2 \end{pmatrix}, v_2 := \begin{pmatrix} -2 \\ 1 \\ 1 \\ 2 \end{pmatrix}, v_3 := \begin{pmatrix} a \\ b \\ 0 \\ d \end{pmatrix}, v_4 := \begin{pmatrix} 2 \\ -5 \\ 7 \\ 6 \end{pmatrix}.$$

   (b) What is the dot product equation of the hyperplane whose direction vectors are v_1, v_2, v_3 and v_4 but which passes through the point $\begin{pmatrix} -1 \\ -3 \\ 2 \\ 6 \end{pmatrix}$? [2]