Cape Breton University

Math 1204

Matrix Algebra

April 2016

Time : 3 hours

Please answer any FIVE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. (a) Considering the set of points (-1,2), (3,1), (-2,0), (-4,4), (4,-1), find the quadratic equation that best fits this data. [7]

(b) (i) Calculate the exact vertical fractional differences between the given values and the quadratic curve and check if they sum to zero. [2]

(ii) Re-use parts of matrices from (a) to determine the best fit line to the data. [1]

(iii) By sketching and/or use of your calculator determine how many points are closer vertically to the best fit line than the quadratic curve. [2]

Q2. (a) Create a matrix A with no zeroes by multiplying a suitably chosen 3×1 matrix B by its transpose. Show that the rank of A is 1, A will have an eigenvalue 0 of multiplicity 2 and find the other eigenvalue. [6]

(b) Determine the eigenvectors of A, use Gram-Schmidt to create orthogonal eigenvectors for the eigenvalue 0 and verify that now all eigenvectors are orthogonal. [4]

(c) Explain why the procedure in (a) will always give a rank 1 matrix if B is $n \times 1$ and not all zeroes. What will the other eigenvalue be for a general B? [2]

Q3. (a) Show that these vectors are not independent and hence find their dependency. [7]

$$\underline{v}_1 := \begin{pmatrix} -2 \\ 2 \\ -2 \\ 1 \end{pmatrix}, \quad \underline{v}_2 := \begin{pmatrix} 2 \\ 1 \\ 3 \\ 2 \end{pmatrix}, \quad \underline{v}_3 := \begin{pmatrix} 3 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad \underline{v}_4 := \begin{pmatrix} 2 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

(b) Check that $\underline{n} := \begin{pmatrix} 2 \\ -3 \\ -3 \\ 4 \end{pmatrix}$ is the normal to the hyperplane with $\underline{v}_1$, $\underline{v}_2$, $\underline{v}_3$ and $\underline{v}_4$ as direction vectors, find the parametric form of the space S which is perpendicular to both $\underline{v}_1$ and $\underline{v}_2$ and find the parameters which show that $\underline{n}$ is in S. [5]

Q4. Two quantities are related by the following recurrences:

$$c_{k+1} := -\frac{94}{3}c_k + 40d_k\ ,\quad d_{k+1} := -24c_k + \frac{92}{3}d_k\ ,\quad c_0 := 99\ ,\quad d_0 := 180$$

(a) Diagonalise the underlying matrix of these equations and hence find a formula for c_n and d_n in terms of powers of the eigenvalues. [9]

(b) What different value of c_0 would ensure that c_n remains positive for all n? Use logarithms to find (for this c_0) the smallest n such that $d_n < 1$. [3]

Q5. (a) Use the rules of matrix algebra to fully simplify this expression for the unknown matrix X, assuming all inverses exist. [8]

$$A(X^T B - 2A^T) = (A - 3B^T)B$$

(b) Check if X only contains small integers using $A := \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ and $B := \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$. Substitute your matrices back into the equation in (a) to see if equality holds. [4]

Q6. (a) Determine which line L in $\mathbb{R}^3$ passes through the points $\begin{pmatrix} 6 \\ 5 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}$ [2]

(b) Let P be the plane with dot product equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix} = 0$. At which point do P and L meet? [3]

(c) Find where P intersects the plane $Q := \begin{pmatrix} 0 \\ -13 \\ 0 \end{pmatrix} + s \times \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} + t \times \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$. Where do L and Q intersect? [7]

Q7. (a) Give reasons why each of the three vector space axioms are true or false for this subset of $\mathbb{R}^2$: (x, y) such that $y \geq x^2$. [4]

(b) Diagonalise $W := \begin{pmatrix} 24 & 20 \\ -15 & -11 \end{pmatrix}$ to get an expression for W^k. Evaluate $W^{\frac{1}{2}}$ using the diagonalisation formula and check that squaring it does give W. [6]

(c) Change the diagonalisation matrix D in (b) to find two other different matrices which will give W when squared. [2]

END OF QUESTION PAPER