Q1 Quadratic is ax^2+bx+c

so

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 4 & -2 & 1 \\ 16 & -4 & 1 \\ 16 & 4 & 1 \end{pmatrix} \qquad \underline{v} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 4 \\ -1 \end{pmatrix}$$

$$A^TA = \begin{pmatrix} 610 & 18 & 46 \\ 18 & 46 & 0 \\ 46 & 0 & 5 \end{pmatrix}$$

$-1+3+-2+-4+4=0$

$1+9+4+16+16=46$

$-1+27-8-64+64=18$

$1+81+16+256+256=610$

$$A^T\underline{v} = \begin{pmatrix} 2+1+0+4-1 \\ -2+3+0-16-4 \\ 2+9+0+64-16 \end{pmatrix} \updownarrow = \cancel{\begin{pmatrix} 6 \\ -19 \\ 59 \end{pmatrix}} \begin{pmatrix} 59 \\ -19 \\ 6 \end{pmatrix}$$

Top was 1st column

Solve

$$\left(\begin{array}{ccc|c} 610 & 18 & 46 & 59 \\ 18 & 46 & 0 & -19 \\ 46 & 0 & 5 & 6 \end{array}\right)$$

$R1 \Leftarrow R1 - 9R3$

$$\left(\begin{array}{ccc|c} 196 & 18 & 1 & 5 \\ 18 & 46 & 0 & -19 \\ 46 & 0 & 5 & 6 \end{array}\right)$$

$R3 \Leftarrow R3 - 5R1$

$$\left(\begin{array}{ccc|c} 196 & 18 & 1 & 5 \\ 18 & 46 & 0 & -19 \\ -934 & -90 & 0 & -19 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 196 & 18 & 1 & 5 \\ 18 & 46 & 0 & -19 \\ -898 & 2 & 0 & -57 \end{array}\right)$$

$R_1 \Leftarrow R_1 - 9R_3 \quad R_2 \Leftarrow R_2 - 23R_3$

$$\left(\begin{array}{ccc|c} 518 & 0 & 1 & 5 \\ 20672 & 0 & 0 & 1292 \\ -898 & 2 & 0 & -57 \end{array}\right)$$

$R_2 \Leftarrow R_2 \times \frac{1}{20672}$

$$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 5/8 \\ 1 & 0 & 0 & 1/16 \\ 0 & 1 & 0 & -7/16 \end{array}\right) \quad = 10/16$$

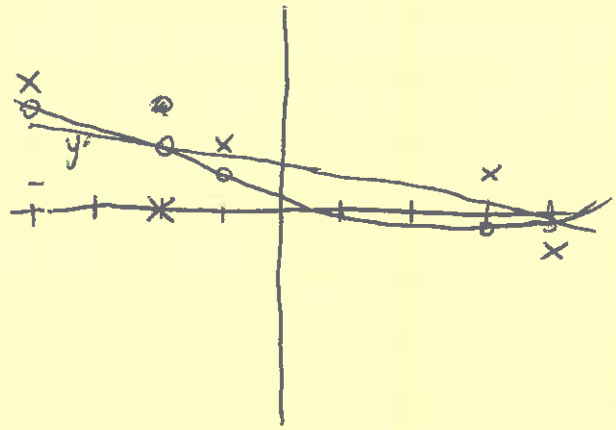
$$y = \frac{x^2 - 7x + 10}{16}$$

Q1

b) Best fit is sol to $\begin{pmatrix} 46 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} -19 \\ 6 \end{pmatrix}$

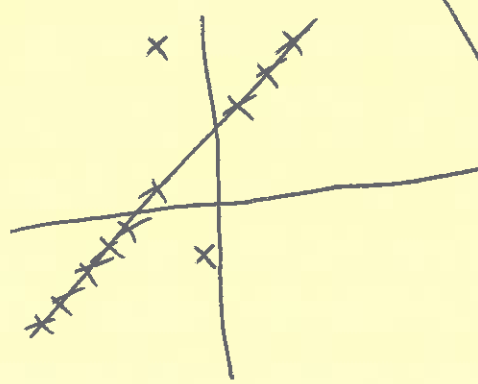
so $m = -\frac{19}{46}$ $c = \frac{6}{5}$ $y'' = -\frac{19}{46}x + \frac{6}{5}$

y''	x_i	y_i	$y' = \frac{x^2-7x+10}{16}$	$y'-y_i$
1.6	-1	$2 = \frac{32}{16}$	$\frac{18}{16}$	$-\frac{14}{16}$
-0.04	3	$1 = \frac{16}{16}$	$-\frac{2}{16}$	$-\frac{18}{16}$
2.0	-2	0	$\frac{28}{16}$	$\frac{28}{16}$
2.9	-4	$4 = \frac{64}{16}$	$\frac{54}{16}$	$-\frac{10}{16}$
-0.5	4	$-1 = -\frac{16}{16}$	$-\frac{2}{16}$	$\frac{14}{16}$
				0



N/A

(we could imagine data like this where all but 2 lie on line and one outlier above and below. A quadratic might get closer to the outliers but won't go through all points exactly?)



for (a) adjoint is $\begin{pmatrix} 27736 & 828 & -2116 \\ 828 & 934 & -90 \\ -2116 & -90 & 230 \end{pmatrix}$

and determinant 41344

Q2) $B=\begin{pmatrix}3\\2\\1\end{pmatrix}$ $B^T=(3\,2\,1)$ $BB^T=\begin{pmatrix}9&6&3\\6&4&2\\3&2&1\end{pmatrix}=A$

Rank$(A)=1$ since $R_2 \Leftarrow R_2-2R_3$ and $R_1\Leftarrow R_1-3R_3$ gives $\begin{pmatrix}0&0&0\\0&0&0\\3&2&1\end{pmatrix}$

$$\det(A-\lambda I)=\det\begin{pmatrix}9-\lambda&6&3\\6&4-\lambda&2\\3&2&1-\lambda\end{pmatrix}\overset{\det}{=}\begin{pmatrix}-\lambda&0&3\lambda\\0&-\lambda&2\lambda\\3&2&1-\lambda\end{pmatrix}=\det\begin{pmatrix}-\lambda&0&0\\0&-\lambda&2\lambda\\3&2&4-\lambda\end{pmatrix}$$
$C_3\Leftarrow C_3+3C_1$

$$=-\lambda\times\det\begin{pmatrix}-\lambda&2\lambda\\2&4-\lambda\end{pmatrix}=-\lambda\times\det\begin{pmatrix}-\lambda&0\\2&6-\lambda\end{pmatrix}=\lambda^2(6-\lambda)$$
$C_2\Leftarrow C_2+2C_1$

So eigenvalues are $\lambda_1=0,\ \lambda_2=0,\ \lambda_3=6$

(b) $3x+2y+z=0$ so $z=-3x-2y$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\0\\-3\end{pmatrix}x+\begin{pmatrix}0\\1\\-2\end{pmatrix}y$

$\underline{v}_1=\begin{pmatrix}1\\0\\-3\end{pmatrix}$ $\underline{v}_2=\begin{pmatrix}0\\1\\-2\end{pmatrix}$

$\underline{e}_1=\begin{pmatrix}1\\0\\-3\end{pmatrix}$ $\underline{e}_2=\begin{pmatrix}0\\1\\-2\end{pmatrix}-\frac{+6}{10}\begin{pmatrix}1\\0\\-3\end{pmatrix}=\frac{1}{5}\left(\begin{pmatrix}0\\5\\-10\end{pmatrix}+\begin{pmatrix}-3\\0\\9\end{pmatrix}\right)$

so $\underline{e}_2=\begin{pmatrix}-3\\5\\-1\end{pmatrix}$ $A\underline{e}_2=\begin{pmatrix}-27+30-3\\-18+20-2\\-9+10-1\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ $\underline{e}_1\cdot\underline{e}_2=-3+0+3=0$

(c) If $B^T=(b_1, b_2 \dots b_n)$ $\det(A-\lambda I)=\lambda^n-(\mathrm{tr}(A))\lambda^{n-1}$

and $\mathrm{tr}(A)=b_1^2+b_2^2+\dots+b_n^2$

Rank 1 since we can do $R_j\Leftarrow R_j-\frac{b_j}{b_1}R_1$ for all $j>1$ to make $00\dots0$

Q3

$$\begin{array}{c} \begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{array} \\ \left(\begin{array}{cccc|c} -2 & 2 & 3 & 2 & 0 \\ 2 & 1 & 1 & -1 & 0 \\ -2 & 3 & 1 & 1 & 0 \\ 1 & 2 & 0 & -1 & 0 \end{array}\right) \end{array}$$

$R1 \Leftarrow R1 - 3R2$
$R3 \Leftarrow R3 - R2$

$$\left(\begin{array}{cccc|c} -8 & -1 & 0 & 5 & 0 \\ 2 & 1 & 1 & -1 & 0 \\ -4 & 2 & 0 & 2 & 0 \\ 1 & 2 & 0 & -1 & 0 \end{array}\right)$$

$R1 \Leftarrow R1 + 5R4$
$R2 \Leftarrow R2 - R4$
$R3 \Leftarrow R3 + 2R4$

$$\left(\begin{array}{cccc|c} -3 & 9 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ -2 & 6 & 0 & 0 & 0 \\ 1 & 2 & 0 & -1 & 0 \end{array}\right)$$

$R3 \Leftarrow R3 \times \frac{1}{2}$ $R4 \Leftarrow R4 \times -1$

$$\left(\begin{array}{cccc|c} -3 & 9 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & -3 & 0 & 0 & 0 \\ -1 & -2 & 0 & 1 & 0 \end{array}\right)$$

$R1 \Leftarrow R1 + 3R3$ $R2 \Leftarrow R2 - R3$ $R4 \Leftarrow R4 + R3$

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 1 & -3 & 0 & 0 & 0 \\ 0 & -5 & 0 & 1 & 0 \end{array}\right)$$

So $\alpha_2 = t$ $\alpha_3 = -2t$
$\alpha_1 = 3t$ $\alpha_4 = 5t$

$$\begin{pmatrix} 3 \\ 1 \\ -2 \\ 5 \end{pmatrix}$$

eg $3\begin{pmatrix} -2 \\ 2 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 3 \\ 1 \\ 1 \\ 0 \end{pmatrix} + 5\begin{pmatrix} 2 \\ -1 \\ 1 \\ -1 \end{pmatrix}$

$$= \begin{pmatrix} -6+2-6+10 \\ 6+1-2-5 \\ -6+3-2+5 \\ 3+2-0-5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(b) $\underline{v}_1 \cdot \underline{n} = -4-6+0+4 = 0$
$\underline{v}_2 \cdot \underline{n} = 4-3-9+8 = 0$
$\underline{v}_3 \cdot \underline{n} = 6-3-3+0 = 0$
$\underline{v}_4 \cdot \underline{n} = 4+3-3-4 = 0$

$$\left(\begin{array}{cccc|c} -2 & 2 & -2 & 1 & 0 \\ 2 & 1 & 3 & 2 & 0 \end{array}\right)$$

$R2 \Leftarrow R2 + R1$

$$\left(\begin{array}{cccc|c} -2 & 2 & -2 & 1 & 0 \\ 0 & 3 & 1 & 3 & 0 \end{array}\right)$$

$R1 \Leftarrow R1 + 2R2$

$$\left(\begin{array}{cccc|c} -2 & 8 & 0 & 7 & 0 \\ 0 & 3 & 1 & 3 & 0 \end{array}\right)$$

$w = 4s + \frac{7t}{2}$
$x = s$
$y = -3s - 3t$
$z = t$

$$\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ -3 \\ 0 \end{pmatrix} s + \begin{pmatrix} 7/2 \\ 0 \\ -3 \\ 1 \end{pmatrix} t$$

$s = -3$ $t = 4$

$$\begin{pmatrix} -12+14 \\ -3+0 \\ 9-12 \\ 0+4 \end{pmatrix} = \underline{n}$$

Q4(a)

$$\begin{pmatrix} c_{k+1} \\ d_{k+1} \end{pmatrix} = \overset{M}{\begin{pmatrix} -94/3 & 40 \\ -24 & 92/3 \end{pmatrix}} \begin{pmatrix} c_k \\ d_k \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} c_k \\ d_k \end{pmatrix} = M^k \begin{pmatrix} 99 \\ 180 \end{pmatrix}$$

$$\det(M-\lambda I) = \left(-\frac{94}{3}-\lambda\right)\left(\frac{92}{3}-\lambda\right) - -24\times 40$$

$$= \lambda^2 + \left(\frac{94}{3}-\frac{92}{3}\right)\lambda - \frac{8648}{9} + 960$$

$$= \lambda^2 + \frac{2}{3}\lambda - \frac{8}{9} = \left(\lambda+\frac{4}{3}\right)\left(\lambda-\frac{2}{3}\right)$$

so $\lambda_1 = -4/3$ $\qquad$ $\lambda_2 = 2/3$

$\underline{v}_1$: $\left(\begin{array}{cc|c} -\frac{90}{3} & 40 & 0 \\ -24 & 90/3 & 0 \end{array}\right)$ $\qquad$ $\underline{v}_2$ $\left(\begin{array}{cc|c} -96/3 & 40 & 0 \\ -24 & 90/3 & 0 \end{array}\right)$

$\left(\begin{array}{cc|c} -30 & 40 & 0 \\ -24 & 32 & 0 \end{array}\right)$ $\qquad$ $\left(\begin{array}{cc|c} -32 & 40 & 0 \\ -24 & 30 & 0 \end{array}\right)$

$R_2 \leftarrow R_2 - \frac{4}{5}R_1$ $\qquad$ $R_2 \leftarrow R_2 - \frac{3}{4}R_1$

$\left(\begin{array}{cc|c} -30 & 40 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\qquad$ $\left(\begin{array}{cc|c} -32 & 40 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$\underline{v}_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\qquad$ $P = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$ $\qquad$ $\underline{v}_2 = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

$$\text{so } M^k \begin{pmatrix} 99 \\ 180 \end{pmatrix} = PD^kP^{-1}\begin{pmatrix} 99 \\ 180 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} (-4/3)^k & 0 \\ 0 & (2/3)^k \end{pmatrix} \frac{1}{16-15} \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 99 \\ 180 \end{pmatrix}$$

$$= \begin{pmatrix} 4\times(-4/3)^k & 5\times(2/3)^k \\ 3\times(-4/3)^k & 4\times(2/3)^k \end{pmatrix} \begin{pmatrix} -504 \\ 423 \end{pmatrix}$$

$$\begin{pmatrix} c_k \\ d_k \end{pmatrix} = \begin{pmatrix} -2016\times(-4/3)^k + 2115\times(2/3)^k \\ -1512\times(-4/3)^k + 1692\times(2/3)^k \end{pmatrix}$$

(b) Need to match $\underline{v}_2$ so $\begin{pmatrix} c \\ 180 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \times t$ $\quad$ $t = \frac{180}{4} = 45$ so $c = 225$

$$\begin{pmatrix} c_k \\ d_k \end{pmatrix} = \begin{pmatrix} 225 \\ 180 \end{pmatrix} \times \left(\frac{2}{3}\right)^k$$

$$\frac{\log 180}{\log 3/2} = 12.8$$

$\underline{k = 13}$

Q5 (a)

$$A\left(X^TB-2A^T\right)=\left(A-3B^T\right)B$$

$\times A^{-1}$ left $\quad A^{-1}A\left(X^TB-2A^T\right)=A^{-1}\left(A-3B^T\right)B$

left inverse distrib $\quad I\left(X^TB-2A^T\right)=\left(A^{-1}A-A^{-1}\left(3B^T\right)\right)B$

left id left inv $\quad X^TB-2A^T=\left(I-3A^{-1}B^T\right)B$

add/right distrib $\quad X^TB=2A^T+B-3A^{-1}B^TB$

right inverse B $\quad X^TBB^{-1}=\left(2A^T+B-3A^{-1}B^TB\right)B^{-1}$

right inverse, distrib $\quad X^TI=2A^TB^{-1}+BB^{-1}-3A^{-1}B^TBB^{-1}$

right identity $\quad X^T=2A^TB^{-1}+I-3A^{-1}B^T$

transpose sum $\quad X=\left(2A^TB^{-1}+I-3A^{-1}B^T\right)^T$

transpose product $\quad =\left(2A^TB^{-1}\right)^T+I^T-\left(3A^{-1}B^T\right)^T$

$$=2\left(B^{-1}\right)^TA+I-3B\left(A^{-1}\right)^T$$

((b) If A is $m\times$ then A^T is $1\times n$ so X must be 1×1, a scalar
Thus $AXA^T=XAA^T$ by scalar distrib)

$A=\begin{pmatrix}2&3\\1&2\end{pmatrix}$ $\quad A^{-1}=\frac{1}{4-3}\begin{pmatrix}2&-3\\-1&2\end{pmatrix}=\begin{pmatrix}2&-3\\-1&2\end{pmatrix}$ $\quad B=\begin{pmatrix}1&1\\3&2\end{pmatrix}$ $\quad B^{-1}=\frac{1}{2-3}\begin{pmatrix}2&-1\\-3&1\end{pmatrix}=\begin{pmatrix}-2&1\\3&-1\end{pmatrix}$

$$X=2\begin{pmatrix}-2&3\\1&-1\end{pmatrix}\begin{pmatrix}2&3\\1&2\end{pmatrix}+\begin{pmatrix}1&0\\0&1\end{pmatrix}-3\begin{pmatrix}1&1\\3&2\end{pmatrix}\begin{pmatrix}2&-1\\-3&2\end{pmatrix}$$

$$=2\begin{pmatrix}-1&0\\1&1\end{pmatrix}+\begin{pmatrix}1&0\\0&1\end{pmatrix}\cancel{+}3\begin{pmatrix}-1&1\\0&1\end{pmatrix}$$

$$=\begin{pmatrix}-2&0\\2&2\end{pmatrix}+\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}3&-3\\0&-3\end{pmatrix}=\begin{pmatrix}2&-3\\2&0\end{pmatrix}\qquad X^T=\begin{pmatrix}2&2\\-3&0\end{pmatrix}$$

$$A\left(X^TB-2A^T\right)=\begin{pmatrix}2&3\\1&2\end{pmatrix}\left(\begin{pmatrix}2&2\\-3&0\end{pmatrix}\begin{pmatrix}1&1\\3&2\end{pmatrix}-\begin{pmatrix}4&2\\6&4\end{pmatrix}\right)=\begin{pmatrix}2&3\\1&2\end{pmatrix}\left(\begin{pmatrix}8&6\\-3&-3\end{pmatrix}-\begin{pmatrix}4&2\\6&4\end{pmatrix}\right)$$

$\left(A-3B^T\right)=\begin{pmatrix}2-3&3-9\\1-3&2-6\end{pmatrix}=\begin{pmatrix}-1&-6\\-2&-4\end{pmatrix}$ $\quad$ so $\left(A-3B^T\right)B=\begin{pmatrix}-1-18&-1-12\\-2-12&-2-8\end{pmatrix}$ $=\begin{pmatrix}2&3\\1&2\end{pmatrix}\begin{pmatrix}4&4\\-9&-7\end{pmatrix}=\begin{pmatrix}-19&-13\\-14&-10\end{pmatrix}$

Q6

(a) direction is $\begin{pmatrix}6\\5\\-4\end{pmatrix}-\begin{pmatrix}0\\2\\5\end{pmatrix}=\begin{pmatrix}6\\3\\-9\end{pmatrix}=3\times\begin{pmatrix}2\\1\\-3\end{pmatrix}$

so line is $\begin{pmatrix}0\\2\\5\end{pmatrix}+t\times\begin{pmatrix}2\\1\\-3\end{pmatrix}$

(ii) Q: $\underline{n}$ is soln to $\left(\begin{array}{ccc|c}1&5&0&0\\-1&-2&1&0\end{array}\right)$ $R_2\Leftarrow R_2+R_1$ $\left(\begin{array}{ccc|c}1&5&0&0\\0&3&1&0\end{array}\right)$ e.g. $\underline{n}=\begin{pmatrix}5\\-1\\3\end{pmatrix}$

so Q has dot prod equation $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}5\\-1\\3\end{pmatrix}=+13$

subs L into P $\begin{pmatrix}2r\\2+r\\5-3r\end{pmatrix}\cdot\begin{pmatrix}5\\-1\\3\end{pmatrix}=+13$ so $10r-2-r+15-9r=+13$

$0r=+13-13=0$

so all r work

L is completely in Q

(b) $\begin{pmatrix}2r\\2+r\\5-3r\end{pmatrix}\cdot\begin{pmatrix}-4\\3\\3\end{pmatrix}=0=-8r+6+3r+15-9r=21-14r$

so $r=\frac{21}{14}=\frac{3}{2}$ and point is $\begin{pmatrix}2\times 3/2\\2+3/2\\5-9/2\end{pmatrix}=\begin{pmatrix}3\\7/2\\1/2\end{pmatrix}$

(c)(i) $\left(\begin{array}{ccc|c}5&-1&3&13\\-4&3&3&0\end{array}\right)$ $R_2\Leftarrow R_2+3R_1$ $\left(\begin{array}{ccc|c}5&-1&3&13\\11&0&12&39\end{array}\right)$

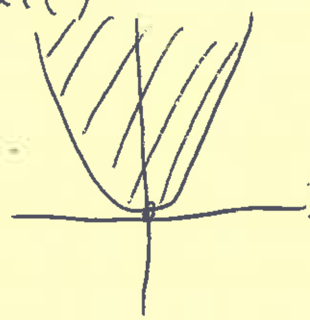
$R_2\Leftarrow R_2\times\frac{1}{11}$ $\left(\begin{array}{ccc|c}5&-1&3&13\\1&0&\frac{12}{11}&\frac{39}{11}\end{array}\right)$

$R_1\Leftarrow R_1-5R_2$ $\left(\begin{array}{ccc|c}0&-1&-\frac{27}{11}&-\frac{52}{11}\\1&0&\frac{12}{11}&\frac{39}{11}\end{array}\right)$

so $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\frac{39}{11}\\\frac{52}{11}\\0\end{pmatrix}+\begin{pmatrix}-\frac{12}{11}\\-\frac{27}{11}\\\frac{11}{11}\end{pmatrix}z$

$=\begin{pmatrix}\frac{39}{11}\\\frac{52}{11}\\0\end{pmatrix}+\begin{pmatrix}-12\\-27\\11\end{pmatrix}\ell$

Q7(a)



$\underline{v} = \begin{pmatrix}1\\2\end{pmatrix}$ is in since $y = 2 \geq 1 = 1^2 = x^2$

$3\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix}$ is not in since $y = 6 \not\geq 9 = 3^2 = x^2$

so axiom 3 is false with $\alpha = 3$ $\underline{v} = \begin{pmatrix}1\\2\end{pmatrix}$

$\begin{pmatrix}0\\0\end{pmatrix}$ is in as $0 \geq 0^2$

A2 true

Similarly axiom 1 is false $\underline{v} = \begin{pmatrix}1\\2\end{pmatrix}$ $\underline{w} = \begin{pmatrix}2\\4\end{pmatrix}$

$\underline{w}$ is in since $y = 4 \geq 2^2 = x^2$

(b) $\det(W - \lambda I) = \lambda^2 - 13\lambda - 264 + 300 = \lambda^2 - 13\lambda + 36 = (\lambda - 9)(\lambda - 4)$

eigenvectors: $W - 9I$: $\left(\begin{array}{cc|c}15 & 20 & 0\\ -15 & -20 & 0\end{array}\right)$ $\underline{v}_1 = \begin{pmatrix}4\\-3\end{pmatrix}$

$W - 4I$: $\left(\begin{array}{cc|c}20 & 20 & 0\\ -15 & -15 & 0\end{array}\right)$ $\underline{v}_2 = \begin{pmatrix}1\\-1\end{pmatrix}$

so $W^k = \begin{pmatrix}4 & 1\\ -3 & -1\end{pmatrix}\begin{pmatrix}9 & 0\\ 0 & 4\end{pmatrix}^k \frac{1}{-4+3}\begin{pmatrix}-1 & -1\\ 3 & 4\end{pmatrix}$

and $W^{1/2} = \begin{pmatrix}4 & 1\\ -3 & -1\end{pmatrix}\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}1 & 1\\ -3 & -4\end{pmatrix} = \begin{pmatrix}12 & 2\\ -9 & -2\end{pmatrix}\begin{pmatrix}1 & 1\\ -3 & -4\end{pmatrix} = \begin{pmatrix}6 & 4\\ -3 & -1\end{pmatrix}$

$\begin{pmatrix}6 & 4\\ -3 & -1\end{pmatrix}\begin{pmatrix}6 & 4\\ -3 & -1\end{pmatrix} = \begin{pmatrix}36-12 & 24-4\\ -18+3 & -12+1\end{pmatrix} = \begin{pmatrix}24 & 20\\ -15 & -11\end{pmatrix}$ ✓

(c) can use $\begin{pmatrix}-3 & 0\\ 0 & -2\end{pmatrix}$ and will get $\begin{pmatrix}-6 & -4\\ 3 & 1\end{pmatrix}$

but also $\begin{pmatrix}3 & 0\\ 0 & -2\end{pmatrix}$ gives $\begin{pmatrix}12 & -2\\ -9 & 2\end{pmatrix}\begin{pmatrix}1 & 1\\ -3 & -4\end{pmatrix} = \begin{pmatrix}12+6 & 12+8\\ -9-6 & -9-8\end{pmatrix} = \begin{pmatrix}18 & 20\\ -15 & -17\end{pmatrix}$

check $\begin{pmatrix}18 & 20\\ -15 & -17\end{pmatrix}\begin{pmatrix}18 & 20\\ -15 & -17\end{pmatrix} = \begin{pmatrix}24 & 20\\ -15 & -11\end{pmatrix}$ ✓