

$$Q1) \begin{pmatrix} w & x & y & z \\ 5 & 2 & -5 & +5 & 5 \\ \textcircled{1} & -2 & 2 & 0 & 0 \\ 1 & 3 & -1 & +1 & 14 \\ -1 & +4 & -5 & +1 & -1 \end{pmatrix}$$

$$\textcircled{R1} \leftarrow \textcircled{R1} - 5\textcircled{R2}$$

$$\textcircled{R3} \leftarrow \textcircled{R3} - \textcircled{R2}$$

$$\textcircled{R4} \leftarrow \textcircled{R4} + \textcircled{R2}$$

$$\begin{pmatrix} 0 & 12 & -15 & 5 & 5 \\ 1 & -2 & 2 & 0 & 0 \\ 0 & 5 & -3 & 1 & 14 \\ 0 & 2 & -3 & \textcircled{1} & -1 \end{pmatrix}$$

$$\textcircled{R1} \leftarrow \textcircled{R1} - 5\textcircled{R4}$$

$$\textcircled{R3} \leftarrow \textcircled{R3} - \textcircled{R4}$$

$$\begin{pmatrix} 0 & 2 & 0 & 0 & 10 \\ 1 & -2 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 & 15 \\ 0 & 2 & -3 & 1 & -1 \end{pmatrix}$$

$$\textcircled{R1} \leftarrow \textcircled{R1} \times \frac{1}{2}$$

$$\textcircled{R3} \leftarrow \textcircled{R3} - \frac{3}{2}\textcircled{R1} \text{ old}$$

$$\textcircled{R2} \leftarrow \textcircled{R2} + \textcircled{R1} \text{ old}$$

$$\textcircled{R4} \leftarrow \textcircled{R4} - \textcircled{R1} \text{ old}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 5 \\ 1 & 0 & 2 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1 & -11 \end{pmatrix}$$

3 pivots so rank=3, one all zero row of 4

$$\textcircled{R1} \quad x = 5$$

$$\textcircled{R2} \quad w + 2y = 10$$

$$\textcircled{R3} \quad 0 = 0$$

$$\textcircled{R4} \quad -3y + z = -11$$

$$\text{let } y = t$$

$$x = 5 + 0 \times t$$

$$y = 0 + 1 \times t$$

$$w = 10 + (-2) \times t$$

$$z = -11 + 3 \times t$$

$$\text{so } \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \\ 0 \\ -11 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \\ 1 \\ 3 \end{pmatrix} t$$

check:

$$\textcircled{R1} \quad 5(10 - 2t) + 2 \times 5 + 5t + 5(-11 + 3t) = 50 - 10t + 10 - 5t - 55 + 15t = 5$$

$$\textcircled{R2} \quad 10 - 2t - 2 \times 5 + 2t = 0$$

$$\textcircled{R3} \quad 10 - 2t + 3 \times 5 - t + (-11 + 3t) = 14 \quad \checkmark$$

$$\textcircled{R4} \quad -(10 - 2t) + 4 \times 5 - 5t - 11 + 3t = -10 + 2t + 20 - 5t - 11 + 3t = -1$$

$$M = \begin{pmatrix} 2 & 7 & 3 & 5 \\ 4 & 14 & 6 & 10 \end{pmatrix} \quad M \begin{pmatrix} 10 \\ 5 \\ 0 \\ -11 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

rank 1 as multiples

$$\textcircled{Q2} \left(\begin{array}{ccc|c} 9 & 3 & 7 & 5 \\ 3 & 7 & 3 & 4 \\ 5 & 3 & 3 & 9 \\ 3 & 5 & 3 & 2 \end{array} \right)$$

$$\textcircled{R2} = \frac{\textcircled{R2} - \textcircled{R4}}{2}$$

$$\left(\begin{array}{ccc|c} 9 & 3 & 7 & 5 \\ 0 & \textcircled{1} & 0 & 1 \\ 5 & 3 & 3 & 9 \\ 3 & 5 & 3 & 2 \end{array} \right)$$

$$\textcircled{R1} = \textcircled{R1} - 3\textcircled{R2}$$

$$\textcircled{R3} = \textcircled{R3} - 3\textcircled{R2}$$

$$\textcircled{R4} = \frac{\textcircled{R4} - 5\textcircled{R2}}{3}$$

$$\left(\begin{array}{ccc|c} 9 & 0 & 7 & 2 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & 3 & 6 \\ 1 & 0 & \textcircled{1} & -1 \end{array} \right)$$

$$\textcircled{R1} = \textcircled{R1} - 7\textcircled{R4}$$

$$\textcircled{R3} = \frac{\textcircled{R3} - 3\textcircled{R4}}{2}$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & 9 \\ 0 & 1 & 0 & 1 \\ \textcircled{1} & 0 & 0 & 9/2 \\ 1 & 0 & 1 & -1 \end{array} \right)$$

$$\textcircled{R1} = \textcircled{R1} - 2\textcircled{R3}$$

$$\textcircled{R4} = \textcircled{R4} - \textcircled{R3}$$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 9/2 \\ 0 & 0 & 1 & -1/2 \end{array} \right)$$

$$\left(\begin{array}{ccc} 9 & 3 & 7 \\ 3 & 7 & 3 \\ 5 & 3 & 3 \\ 3 & 5 & 3 \end{array} \right) \begin{pmatrix} 9/2 \\ 1 \\ -1/2 \end{pmatrix} = \begin{pmatrix} 81/2 + 3 - 77/2 \\ 27/2 + 7 + 33/2 \\ 45/2 + 3 - 33/2 \\ 27/2 + 5 - 33/2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 9 \\ 2 \end{pmatrix}$$