Test 2 2016

Q1) $$\mathrm{Adj}(M) = \begin{pmatrix} \det\begin{pmatrix} 2 & x \\ 2 & -1 \end{pmatrix} & -\det\begin{pmatrix} 2 & x \\ 1 & -1 \end{pmatrix} & \det\begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix} \\ -\det\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} & \det\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} & -\det\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} \\ \det\begin{pmatrix} -1 & 2 \\ 2 & x \end{pmatrix} & -\det\begin{pmatrix} 1 & 2 \\ 2 & x \end{pmatrix} & \det\begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix} \end{pmatrix}^T$$

$$= \begin{pmatrix} -2-2x & 2+x & 2 \\ 3 & -3 & -3 \\ -4-x & 4-x & 4 \end{pmatrix}^T$$

$$= \begin{pmatrix} -2x-2 & 3 & -4-x \\ 2+x & -3 & 4-x \\ 2 & -3 & 4 \end{pmatrix}$$

$$\mathrm{Adj}(M) \times M = \begin{pmatrix} -2x-2+6-4-x & 2x+2+6-8-2x & -4x-4+3x+4+x \\ 2+x-6+4-x & -2-x-6+8-2x & 4+2x-3x-4+x \\ 2-6+4 & -2-6+8 & 4-3x-4 \end{pmatrix}$$

$$= \begin{pmatrix} -3x & 0 & 0 \\ 0 & -3x & 0 \\ 0 & 0 & -3x \end{pmatrix} = -3x I$$

so $\det(M) = -3x$

b) $C_2 \Leftrightarrow C_3$

row 1

$$P = \begin{pmatrix} 1 & 2 & -1 \\ 2 & x & 2 \\ 1 & -1 & 2 \end{pmatrix} \qquad \begin{aligned} \det(P) &= 2x+2-2(4-2)-1(-2-x) \\ &= 2x+2-4+2+x \\ &= 3x \end{aligned}$$

$$Q = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 2 & x \\ -1 & -2 & 1 \end{pmatrix} \qquad \begin{aligned} \det(Q) &= 2+2x+2+x+2(-4+2) \\ &= 4+3x-4 = 3x \end{aligned}$$

c) A row switch changes the determinant's sign, and multiplying a row by α multiplies it by -1, those 2 are the same
M-P will have all columns apart from the 2 switched as zeros and the two switched will be negatives of each other. Thus just 1 pivot will be possible (unless the 2 swapped cols are identical).
Thus rank is 1 (or 0 if the swapped " " ")

Q2) $\left(\begin{array}{ccc|ccc} 3 & 5 & -1 & 1 & 0 & 0 \\ 1 & 4 & -1 & 0 & 1 & 0 \\ 5 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$

$R1 \Leftarrow R1 - R2$

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & -1 & 0 \\ 1 & 4 & -1 & 0 & 1 & 0 \\ 5 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$R2 \Leftarrow R2 - 4R1$

$R3 \Leftarrow R3 - 2R1$

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & -1 & 0 \\ -7 & 0 & -1 & -4 & 5 & 0 \\ 1 & 0 & 0 & -2 & 2 & 1 \end{array}\right)$$

$R1 \Leftarrow R1 - 2R3$

$R2 \Leftarrow R2 + 7R3$

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 5 & -5 & -2 \\ 0 & 0 & -1 & -18 & 19 & 7 \\ 1 & 0 & 0 & -2 & 2 & 1 \end{array}\right)$$

$R1$ to top $R2 \Leftarrow -R2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 1 \\ 0 & 1 & 0 & 5 & -5 & -2 \\ 0 & 0 & 1 & 18 & -19 & -7 \end{array}\right)$$

Check:

$$\begin{pmatrix} 3 & 5 & -1 \\ 1 & 4 & -1 \\ 5 & 2 & 0 \end{pmatrix}\begin{pmatrix} -2 & 2 & 1 \\ 5 & -5 & -2 \\ 18 & -19 & -7 \end{pmatrix}$$

$$= \begin{pmatrix} -6+25-18 & 6-25+19 & 3-10+7 \\ -2+20-18 & 2-20+19 & 1-8+7 \\ -10+10 & 10-10 & 5-4 \end{pmatrix}$$

$= I$ ✓