Math1204 Test 3

March 7^{th}, 2016

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue. The parts of the questions are weighted as shown and can be answered in any order. If you get stuck on a part of a question, ask me and I can give you a hint in return for a mark.

1. (a) Use diagonalisation to find the general expression for A^n if $A := \begin{pmatrix} -66 & -40 \\ 104 & 63 \end{pmatrix}$. [6]

   (b) Give a non-zero $\underline{v}$ such that the entries in $A^n\underline{v}$ are at most 40 for any integer n. [1]

2. We want to use diagonalisation to solve this recurrence:

$$b_{j+1} := 2b_j + 9b_{j-1} - 18b_{j-2} \ , \quad b_0 := 26 \ , \quad b_1 := 19 \ , \quad b_2 := 89$$

   (a) Give the underlying matrix M and the polynomial that its eigenvalues must satisfy. [2]

   (b) Find the eigenvalues by trial substitution, give the eigenvectors and hence M's diagonalisation matrices D and P. [3]

   (c) Determine the correct power of M relating a matrix with b_k in to one with b_0 in. Determine the formula for b_k in terms of powers of the eigenvalues by multiplying out the diagonalisation matrices or solving a matrix equation to avoid having to calculate the inverse of P. [6]

   (d) Use logarithms to find which value of k has b_k first less than -10^{10}. [2]