Test 4 2016

Q1) $A = \begin{pmatrix} -5 & 1 \\ -2 & 1 \\ -1 & 1 \\ 3 & 1 \\ 4 & 1 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} 5 \\ 6 \\ 4 \\ 2 \\ 1 \end{pmatrix}$

$$A^T A = \begin{pmatrix} 25+4+1+9+16 & -5-2-1+3+4 \\ -1 & 1+1+1+1+1 \end{pmatrix} = \begin{pmatrix} 55 & -1 \\ -1 & 5 \end{pmatrix}$$

$$A^T \underline{v} = \begin{pmatrix} -25-12-4+6+4 \\ 5+6+4+2+1 \end{pmatrix} = \begin{pmatrix} -31 \\ 18 \end{pmatrix}$$

$$\begin{pmatrix} m \\ b \end{pmatrix} = \frac{1}{274} \begin{pmatrix} 5 & +1 \\ +1 & 55 \end{pmatrix} \begin{pmatrix} -31 \\ 18 \end{pmatrix} = \frac{1}{274} \begin{pmatrix} -155+18 \\ -31+990 \end{pmatrix} = \frac{1}{274} \begin{pmatrix} -137 \\ +959 \end{pmatrix}$$

so $m = -1/2$ $b = +7/2$

$$y' = \frac{7-x}{2}$$

x_i	-5	-2	-1	3	4
y_i	5	6	4	2	1
y'	6	$9/2$	4	2	$3/2$
$y_i - y'$	-1	$3/2$	0	0	$-1/2$

$\Sigma = 0$

$\overline{(x,y)} = (-1,4)$ and $(3,2)$ are on the line

At most $n-2$ could be on line, $n-1$ isn't possible as sum of diff $\neq 0$

Q2) Since x_i and $-x_i$ are in we have $\sum x_i = \sum x_i^3 = \sum x_i^5 = 0$

$\sum x_i^0 = 4 \quad \sum x_i^2 = 4+1+1+4 = 10 \quad \sum x_i^4 = 16+1+1+16 = 34$

$\sum x_i^6 = 64+1+1+64 = 130$

$\sum y_i = 2+6+9+10 = 27$

$\sum x_i y_i = -4-6+9+20 = 19$

$\sum x_i^2 y_i = 8+6+9+40 = 63 \qquad y = ax^3 + bx^2 + cx + d$

$\sum x_i^3 y_i = -16-6+9+80 = 67$

So
$$\left(\begin{array}{cccc|c} 130 & 0 & 34 & 0 & 67 \\ 0 & 34 & 0 & 10 & 63 \\ 34 & 0 & 10 & 0 & 19 \\ 0 & 10 & 0 & 4 & 27 \end{array}\right)$$

Is equivalent to $\left(\begin{array}{cc|c} 130 & 34 & 67 \\ 34 & 10 & 19 \end{array}\right)$ for $\begin{pmatrix} a \\ c \end{pmatrix} = \frac{1}{1300-34^2}\begin{pmatrix} 10 & -34 \\ -34 & 130 \end{pmatrix}\begin{pmatrix} 67 \\ 19 \end{pmatrix}$

and $\left(\begin{array}{cc|c} 34 & 10 & 63 \\ 10 & 4 & 27 \end{array}\right)$ for $\begin{pmatrix} b \\ d \end{pmatrix} = \frac{1}{136-100}\begin{pmatrix} 4 & -10 \\ -10 & 34 \end{pmatrix}\begin{pmatrix} 63 \\ 27 \end{pmatrix}$

$$\begin{pmatrix} a \\ c \end{pmatrix} = \frac{1}{144}\begin{pmatrix} 670-646 \\ -2278+2470 \end{pmatrix} = \begin{pmatrix} 1/6 \\ 4/3 \end{pmatrix}$$

and
$$\begin{pmatrix} b \\ d \end{pmatrix} = \frac{1}{36}\begin{pmatrix} 252-270 \\ -630+918 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 8 \end{pmatrix}$$

so $y = \frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{4}{3}x + 8 = \frac{x^3 - 3x^2 + 8x + 48}{6}$

Exact fit would be 4x4 so would need around 10 row operations.

Q2(a) $\left(\begin{array}{ccc|c} 34 & 0 & 10 & 63 \\ 0 & 10 & 0 & 19 \\ 10 & 0 & 4 & 27 \end{array}\right)$ $y = px^2 + qx + r$

$$\begin{pmatrix} 34 & 10 \\ 10 & 4 \end{pmatrix}\begin{pmatrix} p \\ r \end{pmatrix} = \begin{pmatrix} 63 \\ 27 \end{pmatrix} \quad \text{so} \quad \begin{pmatrix} p \\ r \end{pmatrix} = \begin{pmatrix} -1/2 \\ 8 \end{pmatrix} \text{ again}$$

and R2 : $10q = 19$

$$y = -\frac{1}{2}x^2 + \frac{19}{10}x + 8 = \frac{-5x^2 + 19x + 80}{10}$$

Row ops for Q2(a)

$$\left(\begin{array}{cccc|c} -8 & 4 & -2 & 1 & 2 \\ -1 & 1 & -1 & 1 & 6 \\ 1 & 1 & 1 & 1 & 9 \\ 8 & 4 & 2 & 1 & 10 \end{array}\right) \quad \begin{array}{l} R1 \Leftarrow \frac{R1 + R4}{2} \\ R2 \Leftarrow \frac{R2 + R3}{2} \end{array} \quad \left(\begin{array}{cccc|c} 0 & 4 & 0 & 1 & 6 \\ 0 & 1 & 0 & 1 & 15/2 \\ 1 & 1 & 1 & 1 & 9 \\ 8 & 4 & 2 & 1 & 10 \end{array}\right)$$

$R4 \Leftarrow \frac{R4 - R1}{2}$

$R3 \Leftarrow R3 - R2$

$\cancel{R1 \Leftarrow R1 - R2}$

$$\cancel{\left(\begin{array}{cccc|c} 0 & 3 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 & 7/2 \\ 1 & 0 & 1 & 0 & 1/2 \\ 4 & 0 & 1 & 0 & 2 \end{array}\right)} \quad \cancel{R1 \Leftarrow R1 \times \frac{1}{3}} \quad \cancel{0\ 1\ 0\ 0\ \ 5/6}$$

$$\left(\begin{array}{cccc|c} 0 & 4 & 0 & 1 & 6 \\ 0 & 1 & 0 & 1 & 15/2 \\ 1 & 0 & 1 & 0 & 3/2 \\ 4 & 0 & 1 & 0 & 2 \end{array}\right) \quad \begin{array}{l} R1 \Leftarrow \frac{R1 - R2}{3} \\ \\ R4 \Leftarrow \frac{R4 - R3}{3} \end{array} \quad \left(\begin{array}{cccc|c} 0 & 1 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & 1 & 15/2 \\ 1 & 0 & 1 & 0 & 3/2 \\ 1 & 0 & 0 & 0 & 1/6 \end{array}\right) \quad \begin{array}{l} b = -1/2 \\ d = 8 \\ c = 4/3 \\ a = 1/6 \end{array}$$