Math1204 Assignment 5: April 2017

Answer all questions and give complete reasons and *checks* for your answers, writing words to explain your conclusions. Please do not erase anything, just put a line through your work and continue; just indicate which answer is your final one.

The questions are weighted as shown and can be answered in any order. Because of everyone having different numbers, do not expect nice round numbers to come as your answers. Use proper fractions for your answers and give any direction vectors using integers.

The numbers represented by a, b, c and d should be replaced by the last four digits of your registration number with any zeros replaced by -1. For instance, if my registration number was 20136502 then i would take $a = 6$, $b = 5$, $c = -1$ and $d = 2$.

Your final solutions should be handed in during the last class on April 6^{th} if you want them back before the final exam (which is on Tuesday 11^{th} starting at 14:00). Otherwise I will hand them back at the exam if they are handed in on April 6^{th}.

1. Let P be the plane $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} -12 \\ d \\ b \end{pmatrix} = -3$ and let L be the line $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d \\ c \\ a \end{pmatrix} + t \times \begin{pmatrix} a \\ 11 \\ c \end{pmatrix}$.

   (a) Determine the point of intersection between P and L by using the dot product form of P. [3]

   (b) What is the line of intersection between P and $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = -5$? [3]

   (c) Solve a matrix equation to show whether or not your line from (b) intersects L. [3]

   (d) Use Gram-Schmidt to get an orthogonal set of direction vectors for P [2]

   (e) Find a parametric form for P with its point and direction vectors with as many zeros in as possible, explaining why you are sure you can't have any more. [3]

2. Consider this hyperplane H in 4 dimensions:

$$\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} a \\ c \\ -1 \\ b \end{pmatrix} = d$$

   (a) Create a line in $\mathbb{R}^4$ which never intersects H and verify that it never meets H. [3]

   (b) Find a plane in $\mathbb{R}^4$ which lies completely within H and verify this. [3]