

Math1204 Algebra Handout 2017

A matrix is of size $m \times n$ (read “ m times n ”) if it has m rows and n columns each containing real numbers. C is square if it is $n \times n$. An $m \times 1$ matrix is called a vector.

Two matrices can only be added or subtracted only if they are of the same size. Two matrices A and B can only be multiplied to form AB if A is of size $m \times n$ and B is $n \times p$. In this case AB will be of size $m \times p$. Recall that, in general, $AB \neq BA$.

The scalar multiple of a matrix $\alpha \times B$ is formed by multiplying all entries of B by the real number α . I is the identity matrix which has zeros everywhere apart from ones on the top left to bottom right diagonal.

For us, only square matrices can have determinants or inverses, although if $\det(C) = 0$ then a square matrix C will not have an inverse, it will be a “singular” matrix.

The following relations are true for any matrices (if they can be multiplied/added/inverted):

$(X + Y) = (Y + X)$ $(X + Y) + Z = X + (Y + Z)$ $X(YZ) = (XY)Z$ $X(Y + Z) = (XY) + (XZ)$ $(X + Y)Z = (XZ) + (YZ)$	Additive Commutativity Additive Associativity Multiplicative Associativity Right Distributivity Left Distributivity
$X(X^{-1}) = I$ $(X^{-1})X = I$ $(X^{-1})^{-1} = X$ $(XY)^{-1} = (Y^{-1})(X^{-1})$	Right Inverse Left Inverse Double Inverse Inverse Product
$(XI) = X$ $(IX) = X$ $X^2 = XX$ $(XY)^2 = XYXY$ $(X + Y)^2 = X^2 + XY + YX + Y^2$	Right Identity Left Identity Matrix Square Square Product Square Sum
$(X + Y)^T = (X^T) + (Y^T)$ $(XY)^T = (Y^T)(X^T)$ $(X^T)^T = X$ $(X^T)^{-1} = (X^{-1})^T$	Transpose Sum Transpose Product Repeated Transpose Transpose Inverse
$(\alpha \times X)Y = \alpha \times (XY)$ $X(\alpha \times Y) = \alpha \times (XY)$ $(\alpha \times X)^T = \alpha \times (X^T)$ $(\alpha \times X)^{-1} = \alpha^{-1} \times (X^{-1})$	Left Scalar Associativity Right Scalar Associativity Scalar Transpose Scalar Inverse ($\alpha \neq 0$)
$\det(XY) = \det(X) \det(Y)$ $\det(X^T) = \det(X)$ $\det(X^{-1}) = (\det(X))^{-1}$ $\det(\alpha X) = (\alpha^n) \det(X)$	Determinant Product Determinant Transpose Determinant Inverse Scalar Determinant (X is $n \times n$)