Cape Breton University

Math 1204

Matrix Algebra

April 2017

Time : 3 hours

Please answer any FIVE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. (a) By doing a column operation followed by a row operation, or by a direct cofactor expansion, find the determinant of $C - \lambda I := \begin{pmatrix} -18-\lambda & -50 & -42 \\ 2 & 3-\lambda & 4 \\ 6 & 20 & 15-\lambda \end{pmatrix}$. [5]

(b) Determine two different eigenvectors of C. Are they orthogonal to each other? [7]

Q2. In this question we are going to be considering this matrix equation;

$$KX^{-1}J = J(K - 2I)$$

(a) What sizes must J and K be if X is $n \times n$? [2]

(b) Solve the equation for X assuming all involved matrices are invertible. [4]

(c) Create invertible 2×2 matrices J and K such that there is no solution to the equation. [4]

(d) Explain why if J or K is not invertible, there can be a solution, and find one. [2]

Q3. (a) Check which of the vector space axioms are true for the subregion of $\mathbb{R}^2$ defined by $y \geq 1 + |2x|$. [4]

(b) Convert the plane $3x + y - z = 2$ to dot product and parametric form. Where does it meet the line $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \times t$? [4]

(c) Find a plane with all of its vector entries as integers which is perpendicular to the plane $\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 5 \\ 5 \end{pmatrix} \times r + \begin{pmatrix} 3 \\ 2 \\ 8 \\ 5 \end{pmatrix} \times s$. [4]

Q4. We have two populations of creatures which are related by the following equations:

$$d_j := 241d_{j-1} - 272e_{j-1}\ , \quad e_j := 210d_{j-1} - 237e_{j-1}\ , \quad d_0 := 89\ , \quad e_0 := 78$$

(a) Use diagonalisation of the underlying matrix to find the general formula for d_k and e_k for any non-negative integer k. [8]

(b) What happens to the populations as time goes by? If instead we had $e_0 = 840$, what value of d_0 would lead to non-increasing populations? [2]

(c) Evaluate the decimal values of the ratios of the terms of the eigenvectors and compare them to the original starting values. Why do these starting values mean that the coefficients of the powers of the eigenvalues are positive? [2]

Q5. (a) Show that these vectors are not independent and write v_1 in terms of v_2 and v_3: [5]

$$v_1 := \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix}\ , \quad v_2 := \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}\ , \quad v_3 := \begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix}$$

(b) Use Gram-Schmidt in the order v_1, v_2 then v_3 and explain why the third vector you get is so simple. [4]

(c) Now use your answers from (b) to use Gram-Schmidt in the order v_1 then v_3 and explain why your second vector is parallel with a vector from (b). [1]

(d) Now apply Gram-Schmidt using one of v_2 or v_3 and then v_1 and explain why your second vectors are not parallel to those from earlier parts of this question. [2]

Q6. (a) Use Row Operations to find the inverse of $Q := \begin{pmatrix} 2 & 1 & 2 & -1 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 1 & -2 \\ 0 & 1 & -1 & 2 \end{pmatrix}$. [8]

(b) Multiply your answer by Q to check if you have any errors, then indicate how all of your row operations in part (a) would have changed the determinant of Q and hence deduce the determinant of Q. [4]

Q7. Suppose we have these data points; (-4,5), (-1,2), (0,0), (2,3), (3,5).

(a) Find the best fit quadratic ($f(x) = ax^2 + bx + c$) for this data, giving a, b and c as fractions. [7]

(b) Determine the vertical distances from your quadratic to the data points and identify which point is furthest from it. [3]

(c) Re-use your working from (a) to give the best fit straight line ($g(x) = mx + d$) and draw a graph showing the data points as well as $f(x)$ and $g(x)$. [2]

END OF QUESTION PAPER