Q1 $\quad C3 \Leftarrow C3 - 2C1$

$$\det(C-\lambda I) = \det\begin{pmatrix} -18-\lambda & -50 & -6+2\lambda \\ 2 & 3-\lambda & 0 \\ 6 & 20 & 3-\lambda \end{pmatrix}$$

$R1 \Leftarrow R1 + 2R3$

$$= \det\begin{pmatrix} -6-\lambda & -10 & 0 \\ 2 & 3-\lambda & 0 \\ 6 & 20 & 3-\lambda \end{pmatrix}$$

$$= (3-\lambda)\times(\lambda^2 - 3\lambda + 6\lambda - 18 + 20)$$

$$= (3-\lambda)(\lambda^2 + 3\lambda + 2)$$

$$= (3-\lambda)(\lambda+1)(\lambda+2)$$

Eigenvalues are $3, -1, -2$

(b) $C - 3I$

$$\left(\begin{array}{ccc|c} -21 & -50 & -42 & 0 \\ 2 & 0 & 4 & 0 \\ 6 & 20 & 12 & 0 \end{array}\right)$$

$R3 \Leftarrow R3 - 3R2 \quad R1 \Leftarrow R1 + \frac{21}{2}R2$

$$\left(\begin{array}{ccc|c} 0 & -50 & 0 & 0 \\ 2 & 0 & 4 & 0 \\ 0 & 20 & 0 & 0 \end{array}\right)$$

$R1 \Leftarrow R1 + \frac{5}{2}R3 \quad R2 \Leftarrow R2 \times \frac{1}{2} \quad R3 \Leftarrow R3/20$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)$$

$$\underline{v}_1 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$C + I$

$$\left(\begin{array}{ccc|c} -17 & -50 & -42 & 0 \\ 2 & 4 & 4 & 0 \\ 6 & 20 & 16 & 0 \end{array}\right)$$

$R3 \Leftarrow R3 - 3R2 \quad R2 \Leftarrow R2 \times \frac{1}{2}$

$R1 \Leftarrow R1 + 17 \text{ new } R2$

$$\left(\begin{array}{ccc|c} 0 & -16 & -8 & 0 \\ 1 & 2 & 2 & 0 \\ 0 & 8 & 4 & 0 \end{array}\right)$$

$R1 \Leftarrow R1 + 2R3 \quad R2 \Leftarrow R2 - \frac{1}{2}R3$

$R3 \Leftarrow R3 \times \frac{1}{4}$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & -2 & 0 & \\ 0 & 2 & 1 & 0 \end{array}\right)$$

$$\underline{v}_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

$C + 2I$

$$\left(\begin{array}{ccc|c} -16 & -50 & -42 & 0 \\ 2 & 5 & 4 & 0 \\ 6 & 20 & 17 & 0 \end{array}\right)$$

$R1 \Leftarrow R1 + 10R2 \quad R3 \Leftarrow R3 - 4R2$

$$\left(\begin{array}{ccc|c} 4 & 0 & -2 & 0 \\ 2 & 5 & 4 & 0 \\ -2 & 0 & 1 & 0 \end{array}\right)$$

$R1 \Leftarrow R1 + 2R3 \quad R2 \Leftarrow R2 - 4R3$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 10 & 5 & 0 & 0 \\ -2 & 0 & 1 & 0 \end{array}\right)$$

$$\underline{v}_3 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

$\underline{v}_1 \cdot \underline{v}_2 = 4 + 0 + 2 = 6$ not

$\underline{v}_1 \cdot \underline{v}_3 = 2 + 0 - 2 = 0$ orthog

$\underline{v}_2 \cdot \underline{v}_3 = 2 - 2 - 4 = -4$ not

(a) If X is $n\times n$ then so is X^{-1} and K is $m\times n$ and J is $n\times l$. K and I are same size so square and JK means $m=l$

$KX^{-1}J$ is $m\times l$, JK is $n\times n$, J is $n\times l$ so $l=n$ and $m=n$

(b) $KX^{-1}J = J(K-2I) = JK-2J$

$KX^{-1} = (JK-2J)J^{-1} = JKJ^{-1}-2I$

$X^{-1} = K^{-1}(JKJ^{-1}-2I) = K^{-1}JKJ^{-1}-2K^{-1}$

$X = (K^{-1}JKJ^{-1}-2K^{-1})^{-1}$ cannot be simplified further

(c) We need $K-2I$ to have no inverse so $K-2I=\begin{pmatrix}1&1\\2&2\end{pmatrix}$ $K=\begin{pmatrix}3&1\\2&4\end{pmatrix}$

$J=\begin{pmatrix}1&-1\\1&0\end{pmatrix}$ $\quad J(K-2I)=\begin{pmatrix}1-2&1-2\\1&1\end{pmatrix}=\begin{pmatrix}-1&-1\\1&1\end{pmatrix}$

So $X^{-1}=K^{-1}\begin{pmatrix}-1&-1\\1&1\end{pmatrix}J^{-1}$

$=\frac{1}{10}\begin{pmatrix}4&-1\\-2&3\end{pmatrix}\begin{pmatrix}-1&-1\\1&1\end{pmatrix}\frac{1}{1}\begin{pmatrix}0&1\\-1&1\end{pmatrix}$

$=\frac{1}{10}\begin{pmatrix}-5&-5\\5&5\end{pmatrix}\begin{pmatrix}0&1\\-1&1\end{pmatrix}=\frac{1}{10}\begin{pmatrix}5&-10\\-5&10\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&-1\\-\frac{1}{2}&1\end{pmatrix}$

But $(X^{-1})^{-1}=X$ and $\begin{pmatrix}\frac{1}{2}&-1\\-\frac{1}{2}&1\end{pmatrix}^{-1}$ doesn't exist as $\det=\frac{1}{2}-\frac{1}{2}=0$

(d) let $J=\begin{pmatrix}1&0\\1&0\end{pmatrix}$ then $X^{-1}J=I^{-1}\begin{pmatrix}1&0\\1&0\end{pmatrix}\begin{pmatrix}-1&0\\0&-1\end{pmatrix}=\begin{pmatrix}-1&0\\-1&0\end{pmatrix}$

$K=\begin{pmatrix}1&0\\0&1\end{pmatrix}=I$

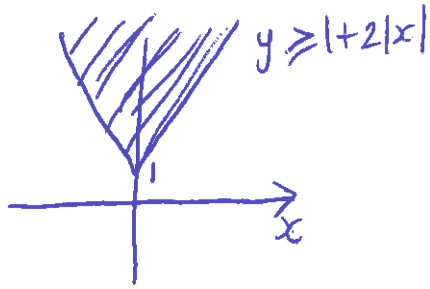
so $X^{-1}=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ $\quad X^{-1}J=\begin{pmatrix}a+b&0\\c+d&0\end{pmatrix}=\begin{pmatrix}-1&0\\-1&0\end{pmatrix}$

So $a=-b-1$ $\quad$ say $b=0$ $\quad X^{-1}=\begin{pmatrix}-1&0\\-2&1\end{pmatrix}$

$d=-c-1$ $\quad c=-2$

$X=\frac{1}{-1}\begin{pmatrix}1&0\\2&-1\end{pmatrix}=\begin{pmatrix}-1&0\\-2&1\end{pmatrix}$ ✓

(a)



Axiom 1 false $\quad y=0 \ngeq 1+2\times|0|=1$

Axiom 2 true

$$y_1 \geq 1+2|x_1|$$
$$y_2 \geq 1+2|x_2|$$
$$y_1+y_2 \geq 2+2|x_1|+2|x_2|$$
$$\geq 2+2|x_1+x_2|$$
$$\geq 1+2|x_1+x_2|$$

Axiom 3 false

$\alpha=-1 \quad \underline{v}=(0,2) \quad 2 \geq 1+2\times 0=1$

$\alpha\underline{v}=(0,-2) \quad -2 \ngeq 1+2\times 0=1$

(b) $\quad 3x+y-z=2 \quad$ is $\begin{pmatrix}3\\1\\-1\end{pmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix}=2$

or $\quad z=3x+y-2 \quad$ so $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\0\\3\end{pmatrix}s+\begin{pmatrix}0\\1\\1\end{pmatrix}t+\begin{pmatrix}0\\0\\-2\end{pmatrix}$

$x=s \quad y=t$

Line is $\begin{pmatrix}2+t\\-1+2t\\3+5t\end{pmatrix}\cdot\begin{pmatrix}3\\1\\-1\end{pmatrix}=2$

$$6+3t-1+2t-3-5t=2$$
$$0t+2=2$$

The line is entirely in the plane

(c)

$$\left(\begin{array}{cccc|c}4&2&5&5&0\\3&2&8&5&0\end{array}\right)$$

$R1 \Leftarrow R1-R2$

$$\left(\begin{array}{cccc|c}1&0&-3&0&0\\3&2&8&5&0\end{array}\right)$$

$R2 \Leftarrow R2-3R1$

$$\left(\begin{array}{cccc|c}1&0&-3&0&0\\0&2&17&5&0\end{array}\right)$$

so $\quad w=3y+0z$

$x=-\frac{17}{2}y-\frac{5}{2}z$

$y=k$

$z=l$

$$\begin{pmatrix}w\\x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\-\frac{17}{2}\\1\\0\end{pmatrix}k+\begin{pmatrix}0\\-\frac{5}{2}\\0\\1\end{pmatrix}l$$

or

$$\begin{pmatrix}w\\x\\y\\z\end{pmatrix}=\begin{pmatrix}6\\-17\\2\\0\end{pmatrix}p+\begin{pmatrix}0\\-5\\0\\2\end{pmatrix}q$$

(a) $M = \begin{pmatrix} 241 & -272 \\ 210 & -237 \end{pmatrix}$

$$\det(M-\lambda I) = (241-\lambda)(-237-\lambda) - 210\times -272$$
$$= \lambda^2 - 4\lambda - 57117 + 57120$$
$$= \lambda^2 - 4\lambda + 3 = (\lambda-3)(\lambda-1)$$

so $\underline{\lambda_1 = 3}$ and $\lambda_2 = 1$

dominant as $|3| > |1|$

$$\begin{pmatrix} d_{j+1} \\ e_{j+1} \end{pmatrix} = M \begin{pmatrix} d_j \\ e_j \end{pmatrix} = \dots = M^{j} \begin{pmatrix} d_0 \\ e_0 \end{pmatrix}$$

$$M^k = P D^k P^{-1}$$

$$D^k = \begin{pmatrix} 3^k & 0 \\ 0 & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 8 & 17 \\ 7 & 15 \end{pmatrix}$$

$$P^{-1} = \frac{1}{120-119} \begin{pmatrix} 15 & -17 \\ -7 & 8 \end{pmatrix} = \begin{pmatrix} 15 & -17 \\ -7 & 8 \end{pmatrix}$$

check $PP^{-1} = \begin{pmatrix} 120-119 & 136-136 \\ 105-105 & -119+120 \end{pmatrix} = I$

$\underline{v}_1$ $\left(\begin{array}{cc|c} 238 & -272 & 0 \\ 210 & -240 & 0 \end{array}\right)$

$R2 \Leftarrow R2 \times \frac{1}{30}$

$\left(\begin{array}{cc|c} 238 & -272 & 0 \\ 7 & -8 & 0 \end{array}\right)$

$R1 \Leftarrow R1 - 34R2$

$\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 7 & -8 & 0 \end{array}\right)$

$\underline{v}_1 = \begin{pmatrix} 8 \\ 7 \end{pmatrix}$

$\underline{v}_2$ $\left(\begin{array}{cc|c} 240 & -272 & 0 \\ 210 & -238 & 0 \end{array}\right)$

$R1 \Leftarrow R1 \div 8$

$R2 \Leftarrow R2 \div 7$

$\left(\begin{array}{cc|c} 30 & -34 & 0 \\ 30 & -34 & 0 \end{array}\right)$

$R2 \Leftarrow R2 - R1$, $R1 \Leftarrow R1 \times \frac{1}{2}$

$\left(\begin{array}{cc|c} 15 & -17 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$\underline{v}_2 = \begin{pmatrix} 17 \\ 15 \end{pmatrix}$

so $$M^k \begin{pmatrix} 89 \\ 78 \end{pmatrix} = \begin{pmatrix} 8 & 17 \\ 7 & 15 \end{pmatrix} \begin{pmatrix} 3^k & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 15 & -17 \\ -7 & 8 \end{pmatrix} \begin{pmatrix} 89 \\ 78 \end{pmatrix}$$

$$= \begin{pmatrix} 8\times 3^k & 17 \\ 7\times 3^k & 15 \end{pmatrix} \begin{pmatrix} 1335-1326 \\ -623+624 \end{pmatrix} = \begin{pmatrix} 72\times 3^k + 17 \\ 63\times 3^k + 15 \end{pmatrix} = \begin{pmatrix} d_k \\ e_k \end{pmatrix}$$

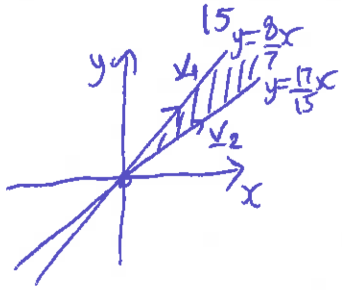
Check $d_0 = 72+17 = 89$ $\quad e_0 = 63+15 = 78$

(b) As $k \to \infty$ both populations go to $+$infinity.

For the dominant eigenvector to not be important we need $\frac{d_0}{840} = \frac{17}{15}$

so $\underline{\underline{d_0 = 952}}$

(c) $\frac{8}{7} = 1.428\dots$ $\quad \frac{17}{15} = 1.1333$ $\quad \frac{89}{78} = 1.141$ which is between them



If we start in shaded area then

$\underline{v}_0 = \alpha \underline{v}_1 + \beta \underline{v}_2$ where α, β are both positive

such as $\frac{89}{78}$

(a)
$$\begin{array}{c} \begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \end{array} \\ \left(\begin{array}{ccc|c} 5 & 3 & 7 & 0 \\ 6 & 2 & 6 & 0 \\ 4 & 2 & 5 & 0 \end{array}\right) \end{array}$$

$R2 \Leftarrow R2 \times \frac{1}{2}$

$$\left(\begin{array}{ccc|c} 5 & 3 & 7 & 0 \\ 3 & 1 & 3 & 0 \\ 4 & 2 & 5 & 0 \end{array}\right)$$

$R1 \Leftarrow R1 - 3R2 \quad R3 \Leftarrow R3 - 2R2$

$$\left(\begin{array}{ccc|c} -4 & 0 & -2 & 0 \\ 3 & 1 & 3 & 0 \\ -2 & 0 & -1 & 0 \end{array}\right)$$

$R2 \Leftarrow R2 + 3R1 \quad R1 \Leftarrow R1 - 2R1$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \end{array}\right)$$

$-3\alpha_1 + \alpha_2 = 0 \quad 2\alpha_1 + \alpha_3 = 0$

so $\alpha_1 = 1 \quad \alpha_2 = 3 \quad \alpha_3 = -2$ works

$\underline{v}_1 + 3\underline{v}_2 - 2\underline{v}_3 = \underline{0}$

or $\underline{v}_1 = -3\underline{v}_2 + 2\underline{v}_3 = \begin{pmatrix} -9+14 \\ -6+12 \\ -6+10 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix}$ ✓

(b) $\underline{e}_1 = \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} \quad \underline{e}_2 : \underline{v}_2 - \dfrac{\underline{v}_2 \cdot \underline{e}_1}{\underline{e}_1 \cdot \underline{e}_2} \underline{e}_1$

$$= \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} - \frac{15+12+8}{25+36+16}\begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} = \frac{11}{11}\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} - \frac{5}{11}\begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix}$$

$$= \frac{1}{11}\left(\begin{pmatrix} 33-25 \\ 22-30 \\ 22-20 \end{pmatrix}\right) = \frac{1}{11}\begin{pmatrix} 8 \\ -8 \\ 2 \end{pmatrix} = \frac{2}{11}\begin{pmatrix} 4 \\ -4 \\ 1 \end{pmatrix}$$

$\underline{e}_2$

$\underline{e}_3 : \underline{v}_3 - \dfrac{\underline{v}_3 \cdot \underline{e}_1}{\underline{e}_1 \cdot \underline{e}_1}\underline{e}_1 - \dfrac{\underline{v}_3 \cdot \underline{e}_2}{\underline{e}_2 \cdot \underline{e}_2}\underline{e}_2$

$$\begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix} - \frac{35+36+20}{77}\begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} - \frac{28-24+5}{16+16+1}\begin{pmatrix} 4 \\ -4 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix} - \frac{13}{11}\begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} - \frac{3}{11}\begin{pmatrix} 4 \\ -4 \\ 1 \end{pmatrix}$$

$$\cancel{= \frac{1}{33}\left(\begin{pmatrix} 231 \\ 198 \\ 165 \end{pmatrix} - \begin{pmatrix} 195 \\ 234 \\ 156 \end{pmatrix} - \begin{pmatrix} 4 \\ -4 \\ 1 \end{pmatrix}\right)}$$

$$= \frac{1}{11}\left(\begin{pmatrix} 77-65-12 \\ 66-78+12 \\ 55-52-3 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

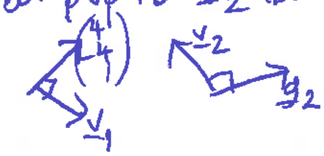
Since they are in a plane the projection is nowhere

(c) $f_1 = \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} \quad f_2 : \underline{v}_3 - \dfrac{\underline{v}_3 \cdot f_1}{f_1 \cdot f_1} f_1 = \begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix} - \dfrac{13}{11}\begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} = \dfrac{3}{11}\begin{pmatrix} 4 \\ -4 \\ 1 \end{pmatrix}$ from before

(d) $g_1 = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} \quad g_2 : \underline{v}_1 - \dfrac{\underline{v}_1 \cdot g_1}{g_1 \cdot g_1}\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} - \dfrac{35}{17}\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} = \dfrac{1}{17}\begin{pmatrix} 85-105 \\ 102-70 \\ 68-70 \end{pmatrix}$

$$= \frac{1}{17}\begin{pmatrix} -20 \\ 32 \\ -2 \end{pmatrix} = \frac{-2}{17}\begin{pmatrix} 10 \\ -16 \\ 1 \end{pmatrix}$$

Now $\begin{pmatrix} 10 \\ -16 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} = 30 - 32 + 2 = 0$ but perp to v_2 not v_1



Q6 (a)

$$\left(\begin{array}{cccc|cccc} 2 & +1 & 2 & -1 & 1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 2 & 0 & 1 & 0 & 0 \\ +1 & 0 & +1 & -2 & 0 & 0 & 1 & 0 \\ 0 & +1 & -1 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$R1 \leftarrow R1 - 2R3$

$$\left(\begin{array}{cccc|cccc} 0 & 1 & 0 & 3 & 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$R4 \leftarrow R4 - R2 \quad R1 \leftarrow R1 - R2$

$$\left(\begin{array}{cccc|cccc} 0 & 0 & 0 & 1 & 1 & -1 & -2 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \end{array}\right)$$

$R2 \leftarrow R2 - 2R1 \quad R3 \leftarrow R3 + 2R1$

$$\left(\begin{array}{cccc|cccc} 0 & 0 & 0 & 1 & 1 & -1 & -2 & 0 \\ 0 & 1 & 0 & 0 & -2 & 3 & 4 & 0 \\ 1 & 0 & 1 & 0 & 2 & -2 & -3 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \end{array}\right)$$

$R3 \leftarrow R3 + R4$

$$\left(\begin{array}{cccc|cccc} 0 & 0 & 0 & 1 & 1 & -1 & -2 & 0 \\ 0 & 1 & 0 & 0 & -2 & 3 & 4 & 0 \\ 1 & 0 & 0 & 0 & 2 & -3 & -3 & 1 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \end{array}\right)$$

R3 to R1 $\quad$ −R4 to R3

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -3 & -3 & 1 \\ 0 & 1 & 0 & 0 & -2 & 3 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & -1 & -2 & 0 \end{array}\right)$$

(b)

$$\begin{pmatrix} 2 & 1 & 2 & -1 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 1 & -2 \\ 0 & 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 & -3 & 1 \\ -2 & 3 & 4 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & -2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4-2-1 & -6+3+2+1 & -6+4+2 & 2-2 \\ -2+2 & 3-2 & 4-4 & 0+0 \\ 2-2 & -3+1+2 & -3+4 & 1-1 \\ -2+2 & 3-1-2 & 4+0-4 & 1 \end{pmatrix}$$

$$= I$$

(c) For the first 6 row operations of the form $R_j \leftarrow R_j + \alpha R_k$ the determinant is unchanged

Swapping twice multiplies twice by -1, and $(-1)^2 = 1$

Multiplying R4 by -1 multiplies the determinant by -1

Since $\det(I) = 1$ we have $\det(Q) = 1 \times (-1)^2 \times -1 = -1$

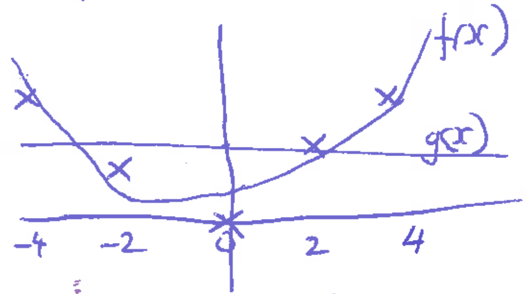
Q7

(a) $A = \begin{pmatrix} 1 & -4 & 16 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$ $\quad \underline{w} = \begin{pmatrix} 5 \\ 2 \\ 0 \\ 3 \\ 5 \end{pmatrix}$

$A^T\underline{w} = \begin{pmatrix} 5+2+0+3+5 \\ -20-2+6+15 \\ 80+2+12+45 \end{pmatrix} = \begin{pmatrix} 15 \\ -1 \\ 139 \end{pmatrix}$

$A^TA = \begin{pmatrix} 5 & 0 & 30 \\ 0 & 30 & -30 \\ 30 & -30 & 354 \end{pmatrix}$

$-4-1+0+2+3=0$
$16+1+0+4+9=30$
$-64-1+8+27=-30$
$256+1+16+81=354$



Solving

$$\left(\begin{array}{ccc|c} 5 & 0 & 30 & 15 \\ 0 & 30 & -30 & -1 \\ 30 & -30 & 354 & 139 \end{array}\right)$$

$R3 \Leftarrow R3 - 6R1 \quad R1 \Leftarrow R1/5$

$$\left(\begin{array}{ccc|c} 1 & 0 & 6 & 3 \\ 0 & 30 & -30 & -1 \\ 0 & -30 & 174 & 49 \end{array}\right)$$

$R3 \Leftarrow R3 + R2 \quad R2 \Leftarrow R2 \times \frac{1}{30}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 6 & 3 \\ 0 & 1 & -1 & -1/30 \\ 0 & 0 & 144 & 48 \end{array}\right)$$

$R3 \Leftarrow R3 \times \frac{1}{144}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 6 & 90/30 \\ 0 & 1 & -1 & -1/30 \\ 0 & 0 & 1 & 10/30 \end{array}\right)$$

$R1 \Leftarrow R1 - 6R3 \quad R2 \Leftarrow R2 + R3$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & +30/30 \\ 0 & 1 & 0 & 9/30 \\ 0 & 0 & 1 & 10/30 \end{array}\right)$$

$a = \frac{+30}{30} \quad b = \frac{9}{30} \quad c = \frac{10}{30}$

$f(x) = \frac{1}{30}(30 + 9x + 10x^2)$

(b) $f(x) - y$

$f(-4) = \frac{1}{30}(30-36+160) = \frac{154}{30} \simeq 5 \qquad \frac{4}{30}$

$f(-1) = \frac{1}{30}(30-9+10) = \frac{31}{30} \simeq 1 \qquad \frac{-29}{30}$

worst

$f(0) = \frac{1}{30}(30+0+0) = 1 \simeq 1 \qquad \frac{30}{30}$

$f(2) = \frac{1}{30}(30+18+40) = \frac{88}{30} \simeq 3 \qquad \frac{-2}{30}$

$f(3) = \frac{1}{30}(30+27+90) = \frac{147}{30} \simeq 5 \qquad \frac{-3}{30}$

$\qquad 0$

(c) $\begin{pmatrix} 5 & 0 \\ 0 & 30 \end{pmatrix}\begin{pmatrix} b \\ m \end{pmatrix} = \begin{pmatrix} 15 \\ -1 \end{pmatrix}$ so $b=3$, $m = -1/30$

$g(x) = \frac{-1}{30}x + 3$