Cape Breton University

Math 1204

MATRIX ALGEBRA

6^{th} March 2017

Time : $\frac{3}{2}$ hours

Please answer any THREE of these questions, and make sure to include all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted. At the end of the examination you are allowed to take the question paper with you and work on any question you haven't already tried and hand it in on Tuesday 7^{th} March in class. The number of marks earned from this take-home question will be divided by 2 and will replace your worst question from the test if it scored lower.

Q1. (a) Solve this equation for X, getting it into the simplest form possible and stating which rules of algebra are being used (you can assume that A is non-singular) [6]

$$5(AXA^{-1} - A) = AB - 3I$$

(b) Why are X and B necessarily the same size as A? Give detailed reasons. [2]

(c) If $A := \begin{pmatrix} 7 & 5 \\ 11 & 8 \end{pmatrix}$ and $B := \begin{pmatrix} 4 & -5 \\ -13 & 6 \end{pmatrix}$, check whether or not your answer for X is made from positive integers less than 9. [4]

Q2. Let us consider this matrix $M := \begin{pmatrix} -32 & -40 & -10 \\ 15 & 18 & 5 \\ 45 & 60 & 13 \end{pmatrix}$

(a) Verify that $\underline{v} := \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ is an eigenvector of M and deduce its eigenvalue. [1]

(b) Use one row and one column operation to create two zeros in a matrix with the same determinant as $M - \lambda I$. [3]

(c) Use the created zeros to get the other eigenvalues of M. [2]

(d) Find the eigenvector of M corresponding to the eigenvalue of multiplicity 1. [3]

(e) Find both eigenvectors of M corresponding to the eigenvalue of multiplicity 2 and find their relation to $\underline{v}$. [3]

Q3. Let $H := \begin{pmatrix} \frac{17}{2} & -6 \\ 10 & -7 \end{pmatrix}$.

(a) Find the eigenvalues and eigenvectors of H. [6]

(b) Use your answer to part (a) to get an expression for H^k for any integer k. [3]

(c) Use your answer to part (b) to evaluate $H^7 \begin{pmatrix} 192 \\ 160 \end{pmatrix}$ and then determine the unique value of c such that $H^k \begin{pmatrix} 192 \\ c \end{pmatrix}$ will head to zero as k increases. [3]

Q4. This will be the matrix for this question:

$$J := \begin{pmatrix} 1 & -3 & 1 \\ 3 & 4 & s \\ 5 & -2 & 2 \end{pmatrix}$$

(a) Show all working and calculate the adjoint of J. Multiply J by your answer to check if it is correct and hence or otherwise find the determinant of J. [6]

(b) (i) Use row operations to solve the matrix equation $J\underline{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ when $s = 8$. [3]

(ii) Why is the value of s almost irrelevant for this question? [1]

(iii) Which value of s would give multiple solutions? [1]

(iv) What would the homogeneous solution be in the case of (iii)? [1]

END OF QUESTION PAPER