Math1204 Algebra Handout 2018

A matrix Z is of size $m \times n$ (read "m times n") if it has m rows and n columns each containing real numbers. The rank of Z is the number of non-zero rows in its reduced row echelon form, and is between 0 and the smaller of m and n. C is a square matrix if it is $n \times n$. An $m \times 1$ matrix is called a (column) vector.

Two matrices can only be added or subtracted only if they are of the same size. Two matrices A and B can only be multiplied to form AB if A is of size $m \times n$ and B is $n \times p$. In this case AB will be of size $m \times p$. Recall that, in general, $AB \neq BA$.

The scalar multiple of a matrix $\alpha \times B$ is formed by multiplying all entries of B by the real number α. I is the identity matrix which has zeros everywhere apart from ones on the top left to bottom right diagonal.

For us, only square matrices can have determinants or inverses, although if $\det(C) = 0$ then a square matrix C will not have an inverse, it will be a "singular" matrix.

The following relations are true for any matrices (if they can be multiplied/added/inverted):

$(X+Y) = (Y+X)$	Additive Commutativity
$(X+Y)+Z = X+(Y+Z)$	Additive Associativity
$X(YZ) = (XY)Z$	Multiplicative Associativity
$X(Y+Z) = (XY)+(XZ)$	Right Distributivity
$(X+Y)Z = (XZ)+(YZ)$	Left Distributivity
$X(X^{-1}) = I$	Right Inverse
$(X^{-1})X = I$	Left Inverse
$(X^{-1})^{-1} = X$	Double Inverse
$(XY)^{-1} = (Y^{-1})(X^{-1})$	Inverse Product (they switch)
$(XI) = X$	Right Identity
$(IX) = X$	Left Identity
$X^2 = XX$	Matrix Square
$(XY)^2 = XYXY$	Square Product
$(X+Y)^2 = X^2+XY+YX+Y^2$	Square Sum
$(X+Y)^T = (X^T)+(Y^T)$	Transpose Sum
$(XY)^T = (Y^T)(X^T)$	Transpose Product (they switch)
$(X^T)^T = X$	Repeated Transpose
$(X^T)^{-1} = (X^{-1})^T$	Transpose Inverse
$(\alpha \times X)Y = \alpha \times (XY)$	Left Scalar Multiple
$X(\alpha \times Y) = \alpha \times (XY)$	Right Scalar Multiple
$(\alpha \times X)^T = \alpha \times (X^T)$	Scalar Transpose
$(\alpha \times X)^{-1} = \alpha^{-1} \times (X^{-1})$	Scalar Inverse ($\alpha \neq 0$)
$\det(XY) = \det(X)\det(Y)$	Determinant Product
$\det(X^T) = \det(X)$	Determinant Transpose
$\det(X^{-1}) = (\det(X))^{-1}$	Determinant Inverse ($\det(X) \neq 0$)
$\det(\alpha X) = (\alpha^n)\det(X)$	Scalar Determinant (X is $n \times n$)