Cape Breton University

Math 1204

MATRIX ALGEBRA

February 2018

Time : $\frac{3}{2}$ hours

Please answer any THREE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted. At the end of the examination you are allowed to take the question paper with you and work on any question you haven't already tried and hand it in by noon on Friday 16$^{\text{th}}$. The number of marks gained from this attempt will be divided by 2 and can replace your lowest mark for a question from the test if it scored lower.

Q1. (a) Determine the adjoint of $M := \begin{pmatrix} 3 & 3 & 1 \\ 6 & -3 & 2 \\ 4 & x & -5 \end{pmatrix}$, showing each step. [5]

(b) Multiply your answer to (a) by M and hence or otherwise get find $\det(M)$. [3]

(c) If $N := \begin{pmatrix} 3 & y & 1 \\ 6 & -3 & 2 \\ 4 & x & -5 \end{pmatrix}$, identify which value of y makes N singular. [3]

Q2. Suppose $B := \begin{pmatrix} -12 & \frac{35}{2} \\ \frac{-15}{2} & 11 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -24 & 35 \\ -15 & 22 \end{pmatrix}$.

(a) Find the eigenvalues of B and determine which one is dominant. [4]

(b) Find the eigenvector of B other than $\begin{pmatrix} 7 \\ 5 \end{pmatrix}$. [3]

(c) Use diagonalisation to get an expression for B^k for any integer k. [3]

(d) If $\underline{u} := \begin{pmatrix} \alpha \\ 75 \end{pmatrix}$, what value of α will mean that $B^k\underline{u}$ tends to $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ as k goes to infinity? [1]

Q3. (a) Use Row Operations to reduce the matrix underlying this system of equations to an equivalent of Reduced Row-Echelon Form. [4]

$$\begin{aligned} 7\,w + 4\,x + 3\,y - z &= 2 \\ 3\,v + 5\,w + 5\,x + 3\,y + z &= 1 \\ 5\,v + 6\,w + 7\,x + 4\,y + 2\,z &= 1 \\ 2\,v + w + 2\,x + y + z &= 0 \end{aligned}$$

(b) Express the solutions to the system of equations in terms of homogeneous and particular solutions which only involve integers. [4]

(c) Give an example of a 4×5 matrix E with rank 1 and no zeroes. Find a way to write E as the product of a 4×1 and 1×5 matrix. Write the matrix of coefficients from part (a) as the product of two rank 2 matrices in a similar way. [3]

Q4. Let $F := \begin{pmatrix} -11 & -20 & 20 \\ 30 & 39 & -30 \\ 15 & 15 & -6 \end{pmatrix}$ for this question.

(a) Show that $\underline{v} := \begin{pmatrix} 9 \\ 1 \\ 10 \end{pmatrix}$ is an eigenvector of both F and F^T with the same eigenvalue μ. [2]

(b) Find a general expression for the eigenvectors of F with the eigenvalue μ. [4]

(c) Repeat (b) for F^T and explain how $\underline{v}$ belongs to the solutions for (b) and (c). [5]

END OF QUESTION PAPER