Q1

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 2 & 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$R_1 \Leftarrow R_1 - R_4$
$R_2 \Leftarrow R_2 - R_4$
$R_3 \Leftarrow R_3 - R_4$

$$\left(\begin{array}{cccc|cccc} -1 & 2 & 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & -1 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$R_1 \Leftarrow R_1 - R_2$
$R_3 \Leftarrow R_3 - 2R_2$

$$\left(\begin{array}{cccc|cccc} -1 & 2 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -2 & 1 & 1 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$R_1 \Leftarrow 2R_3 - R_1$ (to make +1)

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & -3 & 2 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -2 & 1 & 1 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$R_4 \Leftarrow R_4 - 2R_1$
$R_2 \Leftrightarrow R_3$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & -3 & 2 & 2 \\ 0 & 1 & 0 & 0 & 0 & -2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 & 6 & -4 & -3 \end{array}\right)$$

Check

$$\begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 1 & 2 & 1 \\ 2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & -3 & 2 & 2 \\ 0 & -2 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 2 & 6 & -4 & -3 \end{pmatrix} = \begin{pmatrix} -1+2 & -3-4+7 & 2+2-4 & 2+2-4 \\ -2+2 & -6+7 & 4-4 & 4-4 \\ -2+2 & -6-2+2+6 & 4+1-4 & 5-2-3 \\ -2+2 & -6+6 & 4-4 & 4-3 \end{pmatrix} = I \checkmark$$

b) $\det\left(\begin{pmatrix} p & q \\ r & s \end{pmatrix}\right) = ps - qr$

we need p and s to be large eg n and qr small eg 1

so $\det\left(\begin{pmatrix} n & 1 \\ 1 & n \end{pmatrix}\right) = n^2 - 1$ is maximum $< n^2$

$\begin{pmatrix} 2 & & \\ & 2 & 1 \\ & 1 & 2 \end{pmatrix}$ gives us $2 \times (4-1) = 6$ but next is subtracted so make it 0

$\begin{pmatrix} 2 & * & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ but that makes a -3 so $\det = 6 + 0 - 3 = 3$

Try more 2s

$\begin{pmatrix} 2 & 1 & \\ 1 & 2 & \\ \cancel{2} & \cancel{2} & 1 \end{pmatrix}$ gives us 3 to start

$\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ $3 - 2\times + 2 + 2\times - 2 = -5$ big, but negative

$R_1 \Leftrightarrow R_2$

$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ $1 \times -3 + -2 \times -2 + 2 \times 2$
$= -3 + 4 + 4 = 5$

Q2 $B = \begin{pmatrix} 4 & 6 & y \\ 0 & x & -2 \\ 2 & -5 & 3 \end{pmatrix}$

(a) column 1

$$\det(B) = 4 \times \det\left(\begin{pmatrix} x & -2 \\ -5 & 3 \end{pmatrix}\right) - 0 \times \sim + 2 \times \det\left(\begin{pmatrix} 6 & y \\ x & -2 \end{pmatrix}\right)$$
$$= 4 \times (3x - 10) + 2 \times (-12 - xy)$$
$$= 12x - 40 - 24 - 2xy$$
$$= 12x - 64 - 2xy$$

Row 2

$$\det(B) = -0 \times \sim + x \times \det\left(\begin{pmatrix} 4 & y \\ 2 & 3 \end{pmatrix}\right) - -2 \times \det\left(\begin{pmatrix} 4 & 6 \\ 2 & -5 \end{pmatrix}\right)$$
$$= x \times (12 - 2y) + 2 \times (-20 - 12)$$
$$= 12x - 2xy - 64$$

(b) we need the coefficient of x to be 0, so $(12-2y)x = 0$

so set $y = 6$ check $\det(B) = 12x - 64 - 12x = -64 \neq 0$

(or $12x - 64 - 2xy = 0$, $(12-2y)x = 64$, $x = \frac{64}{12-2y} = \frac{32}{6-y}$.) $y \neq 6$

((c) If $y = -2$ then $x = \frac{32}{6--2} = \frac{32}{8} = 4$)

(d) $y = 2$ $x = 8$

$$\left(\begin{array}{ccc|c} 4 & 6 & 2 & 0 \\ 0 & 8 & -2 & 0 \\ 2 & -5 & 3 & 0 \end{array}\right)$$

$R_1 \leftarrow R_1 + R_2$

$R_3 \leftarrow R_3 + \frac{3}{2} R_2$

$$\left(\begin{array}{ccc|c} 4 & 14 & 0 & 0 \\ 0 & 8 & -2 & 0 \\ 2 & 7 & 0 & 0 \end{array}\right)$$

$R_1 \leftarrow R_1 - 2 \times R_3$

$R_1 \leftarrow R_1 \times -\frac{1}{2}$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -4 & 1 & 0 \\ 2 & 7 & 0 & 0 \end{array}\right)$$

so $y = t$

$z = 4t$

$x = -\frac{7}{2} t$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -7/2 \\ 1 \\ 4 \end{pmatrix} t = \begin{pmatrix} -7 \\ 2 \\ 8 \end{pmatrix} \frac{t}{2}$$

check

$$\begin{pmatrix} 4 & 6 & 2 \\ 0 & 8 & -2 \\ 2 & -5 & 3 \end{pmatrix} \begin{pmatrix} -7 \\ 2 \\ 8 \end{pmatrix} = \begin{pmatrix} -28+12+16 \\ 0+16-16 \\ -14-10+24 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$