

Math1204 Test 3

February 7th, 2018

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

1. We are working with this matrix:

$$M := \begin{pmatrix} -3 & 2 & 2 \\ -2 & 9 & 6 \\ 0 & -8 & -5 \end{pmatrix}$$

- (a) Show that $\underline{v}_1 := \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ is an eigenvector of M and deduce its eigenvalue. [2]
- (b) Use a co-factor expansion to get $p(\lambda) := \det(M - \lambda I)$ as a cubic polynomial. [4]
- (c) Use long division to factor $p(\lambda)$ and hence or otherwise determine all eigenvalues of M . [2]
- (d) Find the eigenvector of M other than \underline{v}_1 [4]
(if you can't get the eigenvalue in (c), ask me for it to do this part)
2. (a) Solve this matrix equation for X , giving all properties used and simplifying as much as possible, stating any assumptions you make: [5]

$$2A(I + X) = (A^T - I)A$$

- (b) If $A := \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$, evaluate your answer for X (which should contain only two-digit integers). [3]