

Math1204 Test 4

March 14th, 2018

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

1. For this question we will be investigating the recurrence:

$$c_{k+2} := 4c_{k+1} + 9c_k - 36c_{k-1}, \quad c_0 := 9, c_1 := 65, c_2 := 74.$$

- (a) Use the recurrence to directly calculate c_3 . [1]
- (b) Find all roots of the equation $x^3 - 4x^2 - 9x + 36$ using trial substitution and long division if necessary. [2]
- (c) Use the roots from (b) to give the eigenvectors of the matrix underlying the recurrence and, with P as the matrix of eigenvectors, solve $P\underline{w} = \begin{pmatrix} 74 \\ 65 \\ 9 \end{pmatrix}$ using row operations. [6]
- (d) Give the general formula for c_k using your \underline{w} from (c), check the values the formula gives for c_3 and c_4 . Explain why, with these eigenvalues, c_4 can be less than c_3 , and which values of k should be considered for trying to find when c_k first goes negative, and identify which c_k is the first negative. [3]

2. (a) Find the best fit straight line to this data using the 2×2 matrix method, giving each step. [5]

x_j	1	-1	-3	5	3
y_j	6	12	14	4	6

- (b) Calculate the values of your best fit line at each of the x_j values and the differences from the corresponding y_j values. Use them to draw the graph and find the smallest integer value of x at which the best fit line becomes negative. [3]