Test 4

(a) $C_3 = 4C_2 + 9C_1 - 36C_0 = 4\times74 + 9\times65 - 36\times9$
$= 296 + 585 - 324$
$= 557$

(b) $x^3 - 4x^2 - 9x + 36$

$x=1$: $1 - 4 - 9 + 36 = 24 \neq 0$

$x=2$: $8 - 16 - 18 + 36 = 10 \neq 0$

$x=3$: $27 - 36 - 27 + 36 = 0$

$x=-3$: $-27 + 36 + 27 + 36 = 0$ ←

$$x-3 \;\overline{)\; x^3 - 4x^2 - 9x + 36 \;}\; = x^2 - x - 12$$

$x^3 - 3x^2$

$-x^2 - 9x$

$-x^2 + 3x$

$-12x + 36$

$-12x + 36$

0

$= (x^2 - 9)(x - 4)$

So $x^2 - x + 12 = (x-4)(x+3)$
$= x^2 - 4x + 3x + 12$ ✓

Roots are $\lambda_1 = 3$, $\lambda_2 = 4$, $\lambda_3 = -3$

(c) $P = \begin{pmatrix} 9 & 16 & 9 \\ 3 & 4 & -3 \\ 1 & 1 & 1 \end{pmatrix}$

$\left(\begin{array}{ccc|c} 9 & 16 & 9 & 74 \\ 3 & 4 & -3 & 65 \\ 1 & 1 & 1 & 9 \end{array}\right)$ $R_1 \leftarrow R_1 - 9R_3$
$R_2 \leftarrow R_2 + 3R_3$

$\left(\begin{array}{ccc|c} 0 & 7 & 0 & -7 \\ 6 & 7 & 0 & 92 \\ 1 & 1 & 1 & 9 \end{array}\right)$ $R_1 \leftarrow R_1 \times \frac{1}{7}$
$R_2 \leftarrow R_2 - 6R_1$
$R_3 \leftarrow R_3 - 1\,R_1$

$\left(\begin{array}{ccc|c} 0 & 1 & 0 & -1 \\ 6 & 0 & 0 & 99 \\ 1 & 0 & 1 & 10 \end{array}\right)$ $R_2 \leftarrow R_2 \times \frac{1}{6}$ $R_3 \leftarrow R_3 - 1\,R_2$

$\left(\begin{array}{ccc|c} 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & \frac{33}{2} \\ 0 & 0 & 1 & -\frac{13}{2} \end{array}\right)$ $W = \begin{pmatrix} 33/2 \\ -1 \\ -13/2 \end{pmatrix}$

d) Even k: $C_k = 10\times3^k - 4^k$
Odd k: $C_k = 23\times3^k - 4^k$
So when is $k\log\left(\frac{4}{3}\right) > \log(10)$
and $k\log\left(\frac{4}{3}\right) > \log(23)$

eg $k \geq 8$ and $k \geq 11$
$C_8 = 74$ $C_{11} = -119923$
$C_{10} = -458086$

(d) $C_k = \frac{33}{2}\times3^k - 1\times4^k - \frac{13}{2}\times(-3)^k$

Check $C_0 = \frac{33}{2} - 1 - \frac{13}{2} = 9$ $C_1 = \frac{99}{2} - 4 + \frac{39}{2} = \frac{130}{2} = 65$

$C_3 = \frac{33}{2}\times27 - 64 + \frac{13}{2}\times27 = 23\times27 - 64 = 621 - 64 = 557$

$C_4 = \frac{33}{2}\times81 - 256 - \frac{13}{2}\times81 = 810 - 256 = 554$

Oscillating ±13 $\to -\infty$ as $k \to \infty$

2)

(a) $M = \begin{pmatrix} 1 & 1 \\ -1 & 1 \\ -3 & 1 \\ 5 & 1 \\ 3 & 1 \end{pmatrix}$ $M^T = \begin{pmatrix} 1 & -1 & -3 & 5 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$ $\underline{y} = \begin{pmatrix} 6 \\ 12 \\ 14 \\ 4 \\ 6 \end{pmatrix}$ $f(x) = ax + b$

$M^T M = \begin{pmatrix} 1+1+9+9+25 & 1-1-3+3+5 \\ 5 & 1+1+1+1+1 \end{pmatrix} = \begin{pmatrix} 45 & 5 \\ 5 & 5 \end{pmatrix}$

$M^T \underline{y} = \begin{pmatrix} 6-12-42+20+18 \\ 6+12+14+4+6 \end{pmatrix} = \begin{pmatrix} -10 \\ 42 \end{pmatrix}$

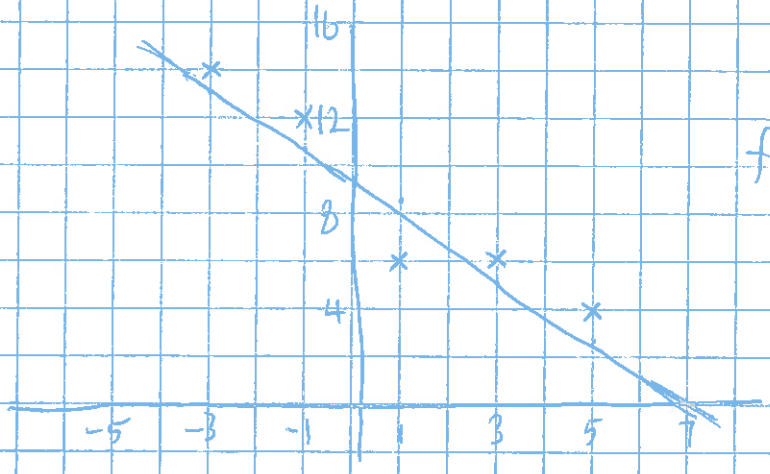
So $\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{200}\begin{pmatrix} 45 & -5 \\ -5 & 45 \end{pmatrix}\begin{pmatrix} -10 \\ 42 \end{pmatrix} = \frac{1}{200}\begin{pmatrix} -50 - 210 \\ 50 + 1890 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} -13 \\ 97 \end{pmatrix}$

So $f(x) = \frac{-13x + 97}{10}$

(b)

x_i	1	-1	-3	5	3
y_i	6	12	14	4	6
$f(x_i)$	8.4	11	13.6	3.2	5.8
$y_i - f(x_i)$	-2.4	1	0.4	0.8	0.2

$\Sigma = 0$



$f(x) < 0$

$\frac{97 - 13x}{10} < 0$

$13x > 97$

$x > \frac{97}{13} = 7.46$

$x \geq 8$