Math1204 Test 5

March 28^{th}, 2018

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

1. Consider a plane P and a line L with the following parametric forms:

$$P : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + j \times \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} + k \times \begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix} \quad , \quad L : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \\ 9 \end{pmatrix} + t \times \begin{pmatrix} 7 \\ 11 \\ 1 \end{pmatrix}$$

   (a) Determine the dot product form of the plane P using row operations. [3]

   (b) Use your answer to (a) to find the point at which P and L intersect. [2]

   (c) Solve a matrix equation to find the values of j and k for the parametric form of P at the point of intersection from (b). [4]

2. Let S be the set of points in the (x, y) plane such that $y \leq x^2$. Show that only one of the three vector subspace axioms is true for S and find counterexamples for the other two axioms. [4]

3. We will be dealing with the following vectors in this question:

$$\underline{v}_1 := \begin{pmatrix} 2 \\ 1 \\ 0 \\ -2 \end{pmatrix} \quad , \quad \underline{v}_2 := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 2 \end{pmatrix} \quad , \quad \underline{v}_3 := \begin{pmatrix} 3 \\ -4 \\ 1 \\ 2 \end{pmatrix}$$

   (a) Answer **just one** of i. or ii. [4]

      i. Prove that $\underline{v}_1$, $\underline{v}_2$ and $\underline{v}_3$ are independent using row operations.
      or

      ii. Determine the dot product equation of the hyperplane H that contains $\underline{v}_1$, $\underline{v}_2$ and $\underline{v}_3$ as direction vectors and passes through the origin.

   (b) Use Gram-Schmidt to find an orthogonal set of vectors using $\underline{v}_1$, $\underline{v}_2$ and $\underline{v}_3$. [3]