University College of Cape Breton

MATH115

INTRODUCTION TO MATRIX ALGEBRA

April 2002
Time : 3 hours

Student Name:
Registration Number:

All questions carry an equal weight and can be attempted in any order but you will choose the **seven** which will give your final mark. Select TWO questions which you do not wish to be counted in your final mark by placing an X in the table below. Failure to comply with this instruction will lead to the 6 **lowest** scoring questions answered being counted! Clearly write your answers to the questions showing all working and checks and indicate what each mathematical calculation is doing. Should the amount of space on the question sheet not be enough, clearly mark PTO on the bottom of the sheet and continue on the other side.

Q1	Q2	Q3	Q4	Q5	Q6	Q7

Q1. (a) Using these two matrices, calculate $X := (A-B)^{-1}$ and evaluate the two matrices AX and BX and note that $AX = BX + I$.

$$A := \begin{bmatrix} 6 & 0 & 1 \\ -7 & -1 & -1 \\ -4 & 2 & 9 \end{bmatrix} \quad B := \begin{bmatrix} -7 & -7 & -3 \\ -7 & -2 & -7 \\ 8 & 5 & -3 \end{bmatrix}$$

(b) Prove this relationship for any matrices A and B for which $A - B$ is invertible, giving your reasons at each stage.

Q2. (a) Carefully find the characteristic polynomial of this matrix:

$$C := \begin{bmatrix} -74 & -219 & -201 \\ 100 & 287 & 264 \\ -76 & -216 & -199 \end{bmatrix}$$

(b) Given that one of the eigenvalues of C is -2, find the other two eigenvalues.

(c) Calculate one of the three eigenvectors and verify you have correctly found it.

Q3. Show that this set of vectors

$$V := \left\{ \begin{pmatrix} 1 \\ -4 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 2 \\ 2 \end{pmatrix} \right\}$$

is independent. Use the Gram Schmidt process to form an orthonormal basis for the space spanned by V.

Q4. Find lower triangular and upper triangular matrices L and U such that

$$LU = \begin{bmatrix} 5 & 5 & -1 \\ -4 & 4 & -8 \\ 8 & 7 & 9 \end{bmatrix}$$

using the LU factorisation method and hence solve this system of equations:

$$\begin{aligned} 5x + 5y - z &= -5 \\ -4x + 4y - 8z &= -2 \\ 8x + 7y + 9z &= -4 \end{aligned}$$

Q5. (a) Use row and column operations and Laplace expansions to evaluate this determinant:

$$\begin{vmatrix} -3 & -2 & 0 & 1 \\ 0 & 1 & a & 1 \\ 1 & 2 & -2 & 1 \\ -5 & -2 & 2 & b \end{vmatrix}$$

(b) Explain why this means that the matrix is non-invertible if $a = -\frac{3}{2}$ or $b = 3$.

(c) Substitute $b = 0$ and $a = -\frac{3}{2}$ into the matrix and find its nullspace.

Q6. By using the matrix curve fitting approximation $M^T MZ = M^T Y$ find the values of d, e and f such that $y = dx^2 + ex + f$ is a least squares best fit for these data points:

x_i	-2	-1	0	2
y_i	4	-1	-2	3

Q7. Use matrix diagonalisation to find the closed form expression for x_i if we are told that $x_{k+2} = 4x_{k+1} - 3x_k$ and $x_0 = 0$ and $x_1 = 4$.

END OF QUESTION PAPER