Math 2301 Handout 1: Row Operations (2019)

- There are three legal row operations which we can use when solving simultaneous equations:

  1. Add a multiple of one row to another row
  2. Multiply a row by a non-zero number
  3. Swap two rows.

  The first of these is the most powerful and will be used most commonly. The second one is only advisable when fractions will not be created, or they are the only way to proceed. The third one is not necessary at all by the below method.

- Given an augmented matrix representing a system of equations we proceed as follows:

  (a) Find a non-zero element on the left side of the dotted line which is as close to zero as possible. It is allowed to use row operations at this stage to make the numbers better if none are what you want.

  (b) Use this small number to make *all* of the numbers in its column (above and below) equal to zero using row operation 1. This is your *pivot*.

  (c) Mark the pivot row and column as used and are not available for use again.

  (d) Return to step (a) unless there are no more non-zero elements left to choose from.

- How to know what the best multiplier in the row operation is?

  If you have a number s in Row j and want to use it to make zero from a t from its column which is in Row k, then you need to do the following row operation:

  $$(\text{Row } k) \leftarrow (\text{Row } k) + \frac{-t}{s} \times (\text{Row } j)$$

  Note that this is the reason why we want our pivot element s to be as small as possible so that $\frac{t}{s}$ is an integer for all other entries in the pivot column.

- What do I do at the end?

  When there are no more columns left unpivoted or no more non-zero entries in rows that haven't already been used you will have an equivalent of Reduced Row Echelon Form. Using this and your original letters you can read off the values of each of the variables in the original equation. If you have columns which are left unpivoted then I recommend giving those variables a new letter as a separate equation.

- Definitions:

  The Rank of a matrix Z is the number of times you can pivot without repeating any columns.

  The difference between the number of columns in a matrix and its rank is the number of independent homogeneous solutions to $Z\underline{v} = \underline{0}$.

Math 2301 Handout 2: Algebra (2019)

A matrix Z is of size $m \times n$ (read "m times n") if it has m rows and n columns each containing real numbers. The rank of Z is the number of non-zero rows in its reduced row echelon form, and is between 0 and the smaller of m and n. C is a square matrix if it is $n \times n$. An $m \times 1$ matrix is called a (column) vector.

Two matrices can only be added or subtracted only if they are of the same size. Two matrices A and B can only be multiplied to form AB if A is of size $m \times n$ and B is $n \times p$. In this case AB will be of size $m \times p$. Recall that, in general, $AB \neq BA$.

The scalar multiple of a matrix $\alpha \times B$ is formed by multiplying all entries of B by the real number α. I is the identity matrix which has zeros everywhere apart from ones on the top left to bottom right diagonal.

For us, only square matrices can have determinants or inverses, although if $\det(C) = 0$ then a square matrix C will not have an inverse, it will be a "singular" matrix.

The following relations are true for any matrices (if they can be multiplied/added/inverted):

$(X+Y) = (Y+X)$	Additive Commutativity
$(X+Y)+Z = X+(Y+Z)$	Additive Associativity
$X(YZ) = (XY)Z$	Multiplicative Associativity
$X(Y+Z) = (XY)+(XZ)$	Right Distributivity
$(X+Y)Z = (XZ)+(YZ)$	Left Distributivity
$X(X^{-1}) = I$	Right Inverse
$(X^{-1})X = I$	Left Inverse
$(X^{-1})^{-1} = X$	Double Inverse
$(XY)^{-1} = (Y^{-1})(X^{-1})$	Inverse Product (they switch)
$(XI) = X$	Right Identity
$(IX) = X$	Left Identity
$X^2 = XX$	Matrix Square
$(XY)^2 = XYXY$	Square Product
$(X+Y)^2 = X^2 + XY + YX + Y^2$	Square Sum
$(X+Y)^T = (X^T)+(Y^T)$	Transpose Sum
$(XY)^T = (Y^T)(X^T)$	Transpose Product (they switch)
$(X^T)^T = X$	Repeated Transpose
$(X^T)^{-1} = (X^{-1})^T$	Transpose Inverse
$(\alpha \times X)Y = \alpha \times (XY)$	Left Scalar Multiple
$X(\alpha \times Y) = \alpha \times (XY)$	Right Scalar Multiple
$(\alpha \times X)^T = \alpha \times (X^T)$	Scalar Transpose
$(\alpha \times X)^{-1} = \alpha^{-1} \times (X^{-1})$	Scalar Inverse ($\alpha \neq 0$)
$\det(XY) = \det(X)\det(Y)$	Determinant Product
$\det(X^T) = \det(X)$	Determinant Transpose
$\det(X^{-1}) = (\det(X))^{-1}$	Determinant Inverse ($\det(X) \neq 0$)
$\det(\alpha X) = (\alpha^n)\det(X)$	Scalar Determinant (X is $n \times n$)