

Q1 (a)

$$\begin{pmatrix} 7 & 5 & 3 & | & 7 \\ 2 & 4 & 3 & | & 8 \\ 3 & 3 & 2 & | & 5 \end{pmatrix}$$

$R_1 \leftrightarrow R_2 - R_2$ (make a 1 at 0)

$$\begin{pmatrix} 5 & 1 & 0 & | & -1 \\ 2 & 4 & 3 & | & 8 \\ -3 & 3 & 2 & | & 5 \end{pmatrix}$$

$R_2 \leftrightarrow R_2 - 4R_1$

$R_3 \leftrightarrow R_3 - 3R_1$

$$\begin{pmatrix} 5 & 1 & 0 & | & -1 \\ -18 & 0 & 3 & | & 12 \\ -12 & 0 & 2 & | & 8 \end{pmatrix}$$

$R_3 \leftrightarrow R_3 \times \frac{1}{2}$

$$\begin{pmatrix} 5 & 1 & 0 & | & -1 \\ -18 & 0 & 3 & | & 12 \\ -6 & 0 & 1 & | & 4 \end{pmatrix}$$

$R_2 \leftrightarrow R_2 - 3R_3$

$$\begin{pmatrix} 5 & 1 & 0 & | & -1 \\ 0 & 0 & 0 & | & 0 \\ -6 & 0 & 1 & | & 4 \end{pmatrix}$$

$$5x + y = -1$$

$$-6x + z = 4$$

$x = t$ unproved

$$y = -1 - 5t$$

$$z = 4 + 6t$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ -5 \\ 6 \end{pmatrix} t$$

partic - homog

$$7x_0 + 5x_1 + 3x_4 = -5 + 12 = 7$$

$$2x_0 + 4x_1 + 3x_4 = -4 + 12 = 8$$

$$3x_0 + 3x_1 + 2x_4 = -3 + 8 = 5$$

$$7x_1 + 5x_5 + 3x_6 = 7 - 25 + 18 = 0$$

$$2x_1 + 4x_5 + 3x_6 = 2 - 20 + 18 = 0$$

$$3x_1 + 3x_5 + 2x_6 = 3 - 15 + 12 = 0$$

(b) $\begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 5 \\ -6 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -2 \end{pmatrix}$

for $x \geq 0$ $t \geq 0$ ~~at $t=0$~~

but then $y = -1 - 5t \leq -1$

Q2

$$\begin{pmatrix} 2 & -1 & 2 & | & 1 \\ -1 & 2 & -1 & | & -1 \\ 2 & 1 & -1 & | & 0 \\ \textcircled{1} & -1 & 1 & | & 0 \end{pmatrix}$$

$$R_1 \leftarrow R_1 - 2R_4$$

$$R_2 \leftarrow R_2 + R_4$$

$$R_3 \leftarrow R_3 - 2R_4$$

$$\begin{pmatrix} 0 & \textcircled{1} & 0 & | & 1 \\ 0 & -1 & 0 & | & -1 \\ 0 & 3 & -3 & | & 1 \\ 1 & -1 & 1 & | & 0 \end{pmatrix}$$

$$R_2 \leftarrow R_2 + R_4$$

$$R_3 \leftarrow R_3 - 3R_4$$

$$R_4 \leftarrow R_4 + R_1$$

$$\begin{pmatrix} 0 & 1 & 0 & | & 1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & -3 & | & -2 \\ 1 & 0 & 1 & | & 1 \end{pmatrix}$$

$$R_3 \leftarrow R_3 + \frac{1}{3}R_2$$

$$\begin{pmatrix} 0 & 1 & 0 & | & 1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & \textcircled{1} & | & \frac{2}{3} \\ 1 & 0 & 1 & | & 1 \end{pmatrix}$$

$$R_4 \leftarrow R_4 - R_3$$

$$\begin{pmatrix} 0 & 1 & 0 & | & 1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 1 & | & \frac{2}{3} \\ 1 & 0 & 0 & | & \frac{1}{3} \end{pmatrix}$$

$$\text{Thus } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1 \\ 2/3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \frac{1}{3} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2-3+4 \\ -1+6-2 \\ 2+3-2 \\ 1-3+2 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 3 \\ 3 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

(b) we did 3 pivots, no more possible as only 3 columns

$$\begin{pmatrix} \textcircled{1} & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$