Test 2 2019

Q1 (a) $\det\left(\begin{pmatrix} 7 & 8 & 2 \\ 4 & 6 & 3 \\ 4 & 3 & -1 \end{pmatrix}\right) = 2\times\det\left(\begin{pmatrix} 4 & 6 \\ 4 & 3 \end{pmatrix}\right)$

$-3\times\det\left(\begin{pmatrix} 7 & 8 \\ 4 & 3 \end{pmatrix}\right)$

$+(-1)\times\det\left(\begin{pmatrix} 7 & 8 \\ 4 & 6 \end{pmatrix}\right)$

$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$

$= 2\times(-12) - 3\times(-11) - 1\times(10) = 33 - 34 = -1$

(b) $\left(\begin{array}{ccc|ccc} 7 & 8 & 2 & 1 & 0 & 0 \\ 4 & 6 & 3 & 0 & 1 & 0 \\ 4 & 3 & -1 & 0 & 0 & 1 \end{array}\right)$

$R_1 \Leftarrow R_1 + 2R_3$

$R_2 \Leftarrow R_2 + 3R_3$

$\left(\begin{array}{ccc|ccc} 15 & 14 & 0 & 1 & 0 & +2 \\ 16 & 15 & 0 & 0 & 1 & 3 \\ 4 & 3 & -1 & 0 & 0 & 1 \end{array}\right)$

$R_1 \Leftarrow R_1 - R_2$

$\left(\begin{array}{ccc|ccc} -1 & -1 & 0 & 1 & -1 & -1 \\ 16 & 15 & 0 & 0 & 1 & 3 \\ 4 & 3 & -1 & 0 & 0 & 1 \end{array}\right)$

$R_2 \Leftarrow R_2 + 15R_1$

$R_3 \Leftarrow R_3 + 3R_1$

$\left(\begin{array}{ccc|ccc} -1 & -1 & 0 & 1 & -1 & -1 \\ 1 & 0 & 0 & 15 & -14 & -12 \\ 1 & 0 & -1 & 3 & -3 & -2 \end{array}\right)$

$R_1 \Leftarrow R_1 + R_2$

$R_3 \Leftarrow R_3 - R_2$

$\left(\begin{array}{ccc|ccc} 0 & -1 & 0 & 16 & -15 & -13 \\ 1 & 0 & 0 & 15 & -14 & -12 \\ 0 & 0 & -1 & -12 & 11 & 10 \end{array}\right)$

$R_1 \Leftrightarrow R_2$

$R_3 \Leftrightarrow -R_3$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 15 & -14 & -12 \\ 0 & 1 & 0 & -16 & 15 & 13 \\ 0 & 0 & 1 & 12 & -11 & -10 \end{array}\right)$

(c) -16 is lowest, comes from

$-1\times\det\left(\begin{pmatrix} 4 & 3 \\ 4 & -1 \end{pmatrix}\right)\times\det(M)$

$= 1\times(-4-12) = -16$

$+16$ is largest in adjoint but determinant is -1 so gives smallest in inverse as inverse = det × adjoint

Check: $\begin{pmatrix} 7 & 8 & 2 \\ 4 & 6 & 3 \\ 4 & 3 & -1 \end{pmatrix}\begin{pmatrix} 15 & -14 & -12 \\ -16 & 15 & 13 \\ 12 & -11 & -10 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ✓

Q2) (a) $2(X+B)A = A+B^T$

right inverse A multiply

$2(X+B)AA^{-1} = (A+B^T)A^{-1}$

inverse distrib

$2(X+B) = AA^{-1} + B^TA^{-1}$

subtract 2B

$2X = I + B^TA^{-1} - 2B$

$\times \frac{1}{2}$ $\left(\frac{1}{2}\times 2 = 1\right)$

$X = \frac{1}{2}\left(I + B^TA^{-1} - 2B\right)$

(b) i) If $A=B$ then $X = \frac{1}{2}\left(I + A^TA^{-1} - 2A\right)$ No simplification

ii) $A=B=\begin{pmatrix} 4 & 7 \\ 3 & 5 \end{pmatrix}$ $A^T = \begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix}$ $A^{-1} = \frac{1}{20-21}\begin{pmatrix} 5 & -7 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} -5 & 7 \\ 3 & -4 \end{pmatrix}$

so $X = \frac{1}{2}\times\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} -5 & 7 \\ 3 & -4 \end{pmatrix} - \begin{pmatrix} 8 & 14 \\ 6 & 10 \end{pmatrix}\right)$

$= \frac{1}{2}\times\left(\begin{pmatrix} -11 & 16 \\ -20 & 29 \end{pmatrix} - \begin{pmatrix} 7 & 14 \\ 6 & 9 \end{pmatrix}\right)$

$= \frac{1}{2}\times\begin{pmatrix} -18 & 2 \\ -26 & 20 \end{pmatrix} = \begin{pmatrix} -9 & 1 \\ -13 & 10 \end{pmatrix}$

iii) $2(X+A)A = A + A = 2A$

so $(X+A)A = A$

or $(X+A-I)A = 0$

so need A with rank 1 eg $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ (Need $A^T = A$)

so $X+A-I$ need to be $C = \begin{pmatrix} 6 & -3 \\ -3 & \frac{3}{2} \end{pmatrix}k$, say so $CA = 0$

$X = C + I - A = \begin{pmatrix} 6k & -3k \\ -3k & \frac{3}{2}k \end{pmatrix} + \begin{pmatrix} 1-1 & 0-2 \\ 0-2 & 1-4 \end{pmatrix} = \boxed{\begin{pmatrix} 6k & -2-3k \\ -2-3k & \frac{3k}{2}-3 \end{pmatrix}}$