Math2301 Test 4 (Recurrences)

March 8^{th}, 2019

Answer all questions and give complete reasons and checks for your answers to earn full marks. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write or for untidyness. The parts of the questions are weighted as shown and can be answered in any order.

If you copy answers, share notes or communicate in any way with another student during the test you will get 0 for this test and will be immediately asked to leave the room.

1. Two populations are linked by the following equations:

$$p_{n+1} := 50p_n - 90q_n \quad , \qquad q_{n+1} := 27p_n - 49q_n$$

   (a) If initial values of $p_0 := 60$ and $q_0 := 33$, use the equations to find q_1 and q_2. [2]

   (b) Identify the matrix underlying the relation and find its eigenvalues and eigenvectors, and get the general formula for p_k and q_k for any integer $k \geq 0$. [6]

   (c) Explain why $p_k > q_k$ for any positive integer k for the given values of p_0 and q_0 and then find the value for p_0 (with $q_0 = 33$) which will ensure that $p_k < q_k$ for all odd k. [2]

2. A recurrence satisfies $a_{n+1} := 12a_n - 29a_{n-1} - 42a_{n-2}$ and $a_0 = -3$, $a_1 = -105$ and $a_2 = -539$.

   (a) Factor the equation underlying the recurrence by showing -1 is a root and doing long division. Hence get the 3 eigenvalues and give the eigenvector matrix P. [3]

   (b) Using the method of diagonalisation, solve $P\underline{w} = \begin{pmatrix} -539 \\ -105 \\ -3 \end{pmatrix}$ to find the general formula for a_k in terms of powers of the eigenvalues. [5]

   (c) Explain why you know the sequence will eventually stop decreasing, and use logarithms to predict at which value a_k will first be positive. [2]

   [The values of a_k will be too big for most calculators, so do not try to give them]