Q1

$$M = \begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \qquad M^TM = \begin{pmatrix} 5 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 34 \end{pmatrix}$$

$$M^T \begin{pmatrix} 2 \\ -5 \\ 3 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 2-5+3+4-1 \\ -4+5+0+4+2 \\ 8-5+4-4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

sum $5, 0, 10, 0, 16+1+0+1+16=34$

so
$$\left(\begin{array}{ccc|c} 5 & 0 & 10 & 3 \\ 0 & 10 & 0 & 3 \\ 10 & 0 & 34 & 3 \end{array}\right)$$

$R3 \Leftarrow R3 - 2R1$
$R2 \Leftarrow R2 \times \frac{1}{10}$

$$\left(\begin{array}{ccc|c} 5 & 0 & 10 & 3 \\ 0 & 1 & 0 & 3/10 \\ 0 & 0 & 14 & -3 \end{array}\right)$$

$R1 \Leftarrow R1 \times 1/5$ $R3 \Leftarrow R3 \times 1/14$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 3/5 \\ 0 & 1 & 0 & 3/10 \\ 0 & 0 & 1 & -3/14 \end{array}\right)$$

$R1 \Leftarrow R1 - 2R3$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 36/35 \\ 0 & 1 & 0 & 3/10 \\ 0 & 0 & 1 & -3/14 \end{array}\right)$$

$$\frac{3}{5} - -\frac{3}{7} = \frac{3 \times 12}{35}$$

so $f(x) = \frac{36}{35} + \frac{3}{10}x - \frac{3}{14}x^2$

$$= \frac{72 + 21x - 15x^2}{70}$$

(b)

x	-2	-1	0	1	2
y	2	-5	3	4	-1
$f(x)$	$\frac{-30}{70}$	$\frac{36}{70}$	$\frac{72}{70}$	$\frac{78}{70}$	$\frac{54}{70}$
$f(x)-y$	$\frac{-170}{70}$	$\frac{+386}{70}$	$\frac{-138}{70}$	$\frac{-202}{70}$	$\frac{124}{70}$

sum is zero

$g(x)$	0	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{9}{10}$	$\frac{12}{10}$
$g(x)-y$	-2	$\frac{53}{10}$	$\frac{-24}{10}$	$\frac{-31}{10}$	$\frac{22}{10}$

(c) $\begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix}\begin{pmatrix} c \\ m \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

so $g(x) = \frac{3}{5} + \frac{3}{10}x$
$= \frac{6 + 3x}{10}$

Equal when
$72 + 21x - 15x^2 = 42 + 21x$
$15x^2 = 30$ $x = \pm\sqrt{2}$

Q2

(a) $\underline{n} \cdot \begin{pmatrix} 6 \\ 0 \\ -5 \end{pmatrix} = \underline{n} \cdot \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = 0$

so $\left(\begin{array}{ccc|c} 6 & 0 & -5 & 0 \\ 3 & 1 & -2 & 0 \end{array}\right)$

$R_1 \leftarrow R_1 \times \frac{1}{5}$ $R_2 \leftarrow R_2 - 2 \times R_1$

$\left(\begin{array}{ccc|c} \frac{6}{5} & 0 & 1 & 0 \\ \frac{25}{5} & 1 & 0 & 0 \end{array}\right)$ $z = \frac{6}{5}x$

$y = -\frac{3}{5}x$

so $\underline{n} = \begin{pmatrix} 5 \\ -3 \\ 6 \end{pmatrix}$

thus ρ is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -3 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -3 \\ 6 \end{pmatrix} = 20 - 15 - 18 = -13$

(b) $\left(\begin{array}{ccc|c} -1 & 3 & 6 & 2 \\ 5 & -3 & 6 & -13 \end{array}\right)$

$R_2 \leftarrow R_2 + 5R_1$

$\left(\begin{array}{ccc|c} -1 & 3 & 6 & 2 \\ 0 & 12 & 36 & -3 \end{array}\right)$

$R_2 \leftarrow R_2 \times \frac{1}{12}$

$\left(\begin{array}{ccc|c} -1 & 3 & 6 & 2 \\ 0 & 1 & 3 & -\frac{1}{4} \end{array}\right)$

$R_1 \leftarrow R_1 - 3R_2$

$\left(\begin{array}{ccc|c} -1 & 0 & -3 & \frac{11}{4} \\ 0 & 1 & 3 & -\frac{1}{4} \end{array}\right)$

so $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \\ 1 \end{pmatrix} t + \begin{pmatrix} \frac{11}{4} \\ -\frac{1}{4} \\ 0 \end{pmatrix}$

If t an integer x, y fractions
If t not, then z is fraction

(c) To not meet it must be parallel to line from (b)

so needs to be say $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} \times t + \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

check:

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3t+2 \\ 3t+t \\ -t \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -3 \\ 6 \end{pmatrix} = 15t - 9t - 6t + 6t$

$= -3t + 3$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 6 \end{pmatrix} = -3t + 2 + 9t - 6t$

$= 2$