# Math-2301 Algebra Handout 2022

A matrix is of size $m \times n$ (read "$m$ times $n$") if it has $m$ rows and $n$ columns each containing real numbers. $C$ is square if it is $n \times n$. An $m \times 1$ matrix is called a vector.

Two matrices can only be added or subtracted only if they are of the same size. Two matrices $A$ and $B$ can only be multiplied to form $AB$ if $A$ is of size $m \times n$ and $B$ is $n \times p$. In this case $AB$ will be of size $m \times p$. Recall that, in general, $AB \neq BA$.

The scalar multiple of a matrix $\alpha \times B$ is formed by multiplying all entries of $B$ by the real number $\alpha$. $I$ is the identity matrix which has zeros everywhere apart from ones on the top left to bottom right diagonal.

For us, only square matrices can have inverses, although a square matrix $C$ might not have an inverse, it will be a "singular" matrix. The thing that will determine whether or not $C$ has an inverse is the determinant of $C$, which we will study soon.

The following relations are true for any matrices (if they can be multiplied/added/inverted):

| | |
|---|---|
| $(X+Y) = (Y+X)$ | Additive Commutativity |
| $(X+Y)+Z = X+(Y+Z)$ | Additive Associativity |
| $X(YZ) = (XY)Z$ | Multiplicative Associativity |
| $X(Y+Z) = (XY)+(XZ)$ | Right Distributivity |
| $(X+Y)Z = (XZ)+(YZ)$ | Left Distributivity |
| $X(X^{-1}) = I$ | Right Inverse |
| $(X^{-1})X = I$ | Left Inverse |
| $(X^{-1})^{-1} = X$ | Double Inverse |
| $(XY)^{-1} = (Y^{-1})(X^{-1})$ | Inverse Product |
| $(XI) = X$ | Right Identity |
| $(IX) = X$ | Left Identity |
| $X^2 = XX$ | Matrix Square |
| $(XY)^2 = XYXY$ | Square Product |
| $(X+Y)^2 = X^2 + XY + YX + Y^2$ | Square Sum |
| $(X+Y)^T = (X^T) + (Y^T)$ | Transpose Sum |
| $(XY)^T = (Y^T)(X^T)$ | Transpose Product |
| $(X^T)^T = X$ | Repeated Transpose |
| $(X^T)^{-1} = (X^{-1})^T$ | Transpose Inverse |
| $(\alpha \times X)Y = \alpha \times (XY)$ | Left Scalar Associativity |
| $X(\alpha \times Y) = \alpha \times (XY)$ | Right Scalar Associativity |
| $(\alpha \times X)^T = \alpha \times (X^T)$ | Scalar Transpose |
| $(\alpha \times X)^{-1} = \alpha^{-1} \times (X^{-1})$ | Scalar Inverse ($\alpha \neq 0$) |
| $\det(XY) = \det(X)\det(Y)$ | Determinant Product |
| $\det(X^T) = \det(X)$ | Determinant Transpose |
| $\det(X^{-1}) = (\det(X))^{-1}$ | Determinant Inverse |
| $\det(\alpha X) = (\alpha^n)\det(X)$ | Scalar Determinant ($X$ is $n \times n$) |