Math 2301 Handout 1: Row Operations (2022)

- There are three legal row operations which we can use when solving simultaneous equations:
  - Add a multiple of one row to another row
  - Multiply a row by a non-zero number
  - Swap two rows.

  The first of these is the most powerful and will be used most commonly. The second one is only advisable when fractions will not be created, or they are the only way to proceed. The third one is not necessary at all by the below method.

- Given an augmented matrix representing a system of equations we proceed as follows:
  1. Find a non-zero element on the left side of the dotted line which is as close to zero as possible. It is allowed to use row operations at this stage to make the numbers better if none are what you want.
  2. Use this small number to make all of the numbers in its column (above and below) equal to zero using the first listed row operation. This is your pivot.
  3. Mark the pivot row and column as used and are not available for use again.
  4. Return to step 1 unless there are no more non-zero elements left to choose from.

- How to know what the best multiplier in the row operation is? If you have a number s in Row j and want to use it to make zero from a t from its column which is in Row k, then you need to do the following row operation:

  $$(\text{Row } k) \leftarrow (\text{Row } k) + \frac{-t}{s} \times (\text{Row } j)$$

  Note that this is the reason why we want our pivot element s to be as small as possible so that $\frac{t}{s}$ is an integer for all other entries in the pivot column.

- What do I do at the end? When there are no more columns left unpivoted or no more non-zero entries in rows that haven't already been used you will have an equivalent of Reduced Row Echelon Form. Using this and your original letters you can read off the values of each of the variables in the original equation. If you have columns which are left unpivoted then I recommend giving those variables a new letter as a separate equation.

- Definitions: The Rank of a matrix Z is the number of times you can pivot without repeating any columns. The difference between the number of columns in a matrix and its rank is the number of independent homogeneous solutions to $Zv = 0$.