Math 2103 Assignment 1: Properties, Independence and Dimension

January 26, 2012

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

1. For this question you will use the matrix M that you have selected and we will be working in $\mathbb{Z}_{13}$ throughout.
   (a) Find the inverse of M using row operations or the adjoint method. [3]
   (b) Determine the two different eigenspaces of M, one is of dimension 2. [4]
   (c) Is there a vector of the form $\begin{pmatrix} a \\ 1 \\ 3 \end{pmatrix}$ for any value of a in your eigenspace? [2]
   (d) Give examples of two other 3×3 matrices for which either all, none or some values of a are in the eigenspaces. (choose matrices which have the two properties that your M doesn't have) [3]

2. Which axioms would our underlying number system need to have for this matrix property to hold? Give the entire proof with reasons for each step. [4]

$$\alpha(AB) = A(\alpha B)$$

3. (a) Verify that the columns of M form an independent set by solving the vanishing equation. Explain why any square matrix with non-zero determinant must have independent columns. [3]
   (b) What is the dimension of the space of polynomials of maximum degree 3 which have both 2 and 5 as roots? Give a basis for this space. [2]
   (c) What is the dimension of the space of 3×3 matrices which satisfy $A^T = 2A$? What is the answer for $n \times n$ matrices in general? [4]

$$\begin{pmatrix} 3 & 7 & 3 \\ 9 & 11 & 4 \\ 6 & 12 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 3 & 11 \\ 8 & 8 & 6 \\ 7 & 10 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 12 & 11 & 8 \\ 8 & 12 & 11 \\ 7 & 11 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 & 10 \\ 9 & 9 & 10 \\ 9 & 2 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 11 & 1 & 1 \\ 6 & 3 & 12 \\ 6 & 12 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 1 & 5 \\ 7 & 8 & 7 \\ 11 & 10 & 2 \end{pmatrix}$$