Math 2103 Assignment 2: Bases and Spaces

February 28, 2012

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

The numbers represented by a, b and c should be replaced by the last three digits of your registration number in that order, and if the digit is 0 it is replaced by 10. For instance, if my registration number was 20015270 then i would take $a = 2$, $b = 7$ and $c = 10$.

1. Create a vector space which is of dimension 2 which contains the vector $x^2 - a$ but doesn't contain any vector in $\text{span}(\{cx^2 + x + 2\})$. Explain how you could create a vector space of dimension $n \geq 3$ which has this property also. Why can't it be done for $n = 2$? [4]

2. T is a linear transformation such that the following holds:

$$T(5x^2+2x+b) := \begin{pmatrix} 2 & -4 \\ 3 & 2 \end{pmatrix}, \quad T(4x^2+2x+3) := \begin{pmatrix} 3 & -8 \\ 8 & 2 \end{pmatrix}, \quad T(2x^2+x+1) := \begin{pmatrix} 0 & -4 \\ 7 & -2 \end{pmatrix}$$

   (a) Find $T(ex^2 + fx + g)$ for general e, f and g. [5]

   (b) What are the kernel and image spaces of T and their dimensions? [4]

   (c) Give two different matrices which aren't in the image space and which aren't multiples of each other. [2]

3. We redefine vector addition and scalar multiplication in $\mathbb{R}^3$ to be the following:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} := \begin{pmatrix} x_1 + x_2 \\ y_1 + z_2 \\ z_1 + y_2 \end{pmatrix}, \quad \alpha \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} := \begin{pmatrix} \alpha x \\ z \\ y \end{pmatrix}$$

   Check whether or not vector space axioms A3, A4, A5, S2, S3 and S4 are true or false, giving counterexamples for the false ones and algebra to prove the true ones. [10]