Math 2103 Assignment 3: Isomorphism and Operators

March 15, 2012

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

The numbers represented by a, b and c should be replaced by the last three digits of your registration number in that order, and if the digit is 0 it is replaced by 10. For instance, if my registration number was 20015270 then i would take $a = 2$, $b = 7$ and $c = 10$.

1. (a) Create a linear operator T in the vector space $\mathbb{P}_2$ by using diagonalisation of a matrix. It should have $x^2 - bx + c$ in its kernel and have $T(ax^2 + 2x - 1) = 2ax^2 + 4x - 2$. [6]

   (b) Find all T-invariant subspaces explaining why you know you have them all. Determine the orthogonal complements of these spaces (using the polynomial equivalent of the dot product definition of orthogonal) and check whether any new spaces are formed and if any are T-invariant. [7]

   (c) Why doesn't T have an inverse operator? Create a non-trivial isomorphism S which has $S(2ax^2 + 4x - 2) = ax^2 + 2x - 1$ and find an expression for $(S \circ T)(ex^2 + fx + g)$. Is this an isomorphism? [5]

2. A matrix M is called idempotent if $M^2 = M$. Solve the equations that must be satisfied for any 2×2 idempotent matrix and hence choose one with no zeroes which nobody else in the class has selected. Use your matrix to form a linear operator R on the complex numbers. Is R reducible? [7]