Math2103 Linear Algebra: Assignment 1 (September 2015)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You can use Maple at any point and can email me any worksheets you created.

You are reminded that plagiarism is a serious offense and when it is detected you will be punished. Feel free to discuss the questions in general with myself and your colleagues but the work attempted must be yours alone.

1. The whole of this question should be done using $\mathbb{Z}_{13}$.
   (a) Given your matrix M find its eigenvalues. [2]
   (b) Determine all of the eigenvectors of M, writing your answers as vector spaces. [4]
   (c) For M's eigenvalue of multiplicity 2 create two spanning eigenvectors with dot product 0 and also a non-zero eigenvector whose dot product with itself is 0. How would this affect the Gram-Schmidt method? Discuss briefly how it can still be used. [4]
2. Determine which properties the underlying number system must have for the $(i, j)^{\text{th}}$ entry of the matrix product $A(B+C)$ to be the same as the $(i, j)^{\text{th}}$ entry of $AB+AC$, showing all details of the calculation. [3]
3. This question should be done using the real numbers $\mathbb{R}$.
   (a) Create an isomorphism T between $\mathbb{R}^3$ and $\mathbb{P}_2$ such that the second column of M maps to $2x^2 + 5x - 3$ and the third column of M maps to $x + 3$, explaining why you are sure it is an isomorphism. [3]
   (b) Find the inverse isomorphism S from $\mathbb{P}_2$ to $\mathbb{R}^3$ in terms of what it maps ax^2+bx+c to and check that it maps back to the columns of M as appropriate. [3]
   (c) Explain why, despite $2x^2+5x-3 = (x+3)(2x-1)$ being true, there is no connection between $S(2x - 1)$ and the S-transformations of the other polynomials. [1]

$$M := \begin{pmatrix} 10 & 1 & 11 \\ 2 & 8 & 7 \\ 6 & 9 & 0 \end{pmatrix} \leftarrow \text{Graeme} \qquad M := \begin{pmatrix} 12 & 9 & 8 \\ 5 & 7 & 11 \\ 2 & 3 & 0 \end{pmatrix} \leftarrow \text{John} - \text{Oliver}$$

$$M := \begin{pmatrix} 1 & 4 & 5 \\ 8 & 6 & 2 \\ 11 & 10 & 0 \end{pmatrix} \leftarrow \text{Joseph} \qquad M := \begin{pmatrix} 3 & 12 & 2 \\ 11 & 5 & 6 \\ 7 & 4 & 0 \end{pmatrix} \leftarrow \text{TJ}$$