Math2103 Linear Algebra: Assignment 2 (Early October 2015)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You can use Maple at any point and can email me any worksheets you created.

You are reminded that plagiarism is a serious offense and when it is detected you will be punished. Feel free to discuss the questions in general with myself and your colleagues but the work attempted must be yours alone. 2^{d-1} marks will be forfeited if you hand your work in d days after the deadline.

1. Let the last 4 digits of your registration number be a, b, c and d for this question. Let T be such that

$$T(\begin{pmatrix} p & q \\ r & s \end{pmatrix}) := (bp - q)x^2 + (q - dr)x + (dr - bp).$$

   (a) Verify that T satisfies either axiom LA1 or LA2. Prove that $\mathrm{Im}(L)$ is a vector subspace for any linear transformation L. [3]

   (b) Determine $\mathrm{Ker}(T)$ and $\mathrm{Im}(T)$, identify their dimensions and that they satisfy the dimension theorem. [4]

   (c) Find a basis set J for $\mathrm{Im}(T)$ with the maximum number of zeroes in, explaining why you believe that has been achieved. [2]

   (d) Find simple matrices K_i such that $T(K_i) = j_i(x)$ for each polynomial $j_i(x)$ in J. Determine $T(\begin{pmatrix} a & b \\ c & d \end{pmatrix})$ and express it in terms of $\sum_i \beta_i j_i(x)$. [3]

   (e) Show that $\begin{pmatrix} a & b \\ c & d \end{pmatrix} - (\sum_i \beta_i K_i)$ is in $\mathrm{Ker}(T)$ as we proved in class. [2]

   (f) Find a linear transformation S such that $T \circ S$ or $S \circ T$ is an isomorphism. [2]

2. For $n \times n$ matrices (please try general n, but at least show for $n = 2$), find the standard bases for the kernel and image of the linear transformation $S(M) := M^T + M$ (that is the sum of M and its transpose). Hence determine the dimensions of the kernel and image. [4]