# Math2103 Linear Algebra: Assignment 3 (Early November 2015)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You can use Maple at any point and can email me any worksheets you created.

You are reminded that plagiarism is a serious offense and when it is detected you will be punished. Feel free to discuss the questions in general with myself and your colleagues but the work attempted must be yours alone. $2^{d-1}$ marks will be forfeited if you hand your work in $d$ days after the deadline. Let the last 3 digits of your registration number be $a$, $b$ and $c$.

1. Let $T$ be the linear operator such that

$$T(\begin{pmatrix} p & q \\ r & s \end{pmatrix}) := \begin{pmatrix} 3p - 6r & 8s \\ cp - 2cr & -2q \end{pmatrix}.$$

   (a) Find the eigenvalues of the underlying $4 \times 4$ matrix and hence identify all of the $T$-invariant subspaces of dimension 1. [2]

   (b) Determine all of the other $T$-invariant subspaces of all other dimensions by combining the different eigenspaces in all possible ways. [4]

2. Define an inner product as follows:

$$\langle f(x), g(x) \rangle := \int_0^1 f(x)g(x)\mathrm{d}x$$

   (a) Find the value of $e$ such that $a + x$ is orthogonal to $e + 3x$. [2]

   (b) Extend these two vectors to make an orthogonal basis set for $\mathbb{P}_3$, express $h(x) := 2x^3 + ax^2 + bx + c$ in terms of this basis, and hence evaluate $||h(x)||$. [5]

3. (a) Using Cholesky, show that your matrix $M$ is not positive definite and find a vector $v$ such that $v^T M v < 0$. Can you change one number in $M$ and make it positive definite? Justify your answer. [4]

   (b) Using diagonalisation, prove that if all eigenvalues of $A$ are positive then $v^T A v$ is positive definite. [3]

$$M := \begin{pmatrix} 12 & 6 & -2 \\ 6 & 39 & -32 \\ -2 & -32 & 27 \end{pmatrix}$$

$$M := \begin{pmatrix} 9 & 10 & 14 \\ 10 & 11 & 14 \\ 14 & 14 & 20 \end{pmatrix}$$

$$M := \begin{pmatrix} 36 & 21 & 9 \\ 21 & 11 & 4 \\ 9 & 4 & 1 \end{pmatrix}$$

$$M := \begin{pmatrix} 16 & -9 & 12 \\ -9 & \frac{9}{2} & -7 \\ 12 & -7 & 9 \end{pmatrix}$$