Math2103 Linear Algebra: Assignment 4 (Late November 2015)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You can use Maple at any point and can email me any worksheets you created.

You are reminded that plagiarism is a serious offense and when it is detected you will be punished. Feel free to discuss the questions in general with myself and your colleagues but the work attempted must be yours alone. 2^{y-1} marks will be forfeited if you hand your work in y days after the deadline. Let the last 4 digits of your registration number be a, b, c and d.

1. By taking an eigenvector $\underline{v}$ of a complex valued matrix M and extending $\underline{v}$ to an orthonormal basis, prove by induction that we can find a unitary matrix U such that $U^*MU = S$ where S is an upper triangular matrix with the eigenvalues on the diagonal. Demonstrate how this works for a small, well chosen (but not Hermitian or symmetric) matrix, unique within the class. Inform me of your matrix choice once it works to make sure it hasn't been previously chosen by another student. [9]

2. Let matrix $L := \begin{pmatrix} 2 & b & c \\ a & d & -1 \end{pmatrix}$.

   (a) Verify that if $J := LL^T$ and $K := L^TL$ then $\det(J - \lambda I)$ and $\det(K - \lambda I)$ are closely related and explain how this will work for an $m \times n$ matrix in general. [3]

   (b) Find the eigenvectors of K and check that for each eigenvector $\underline{w}$ that $L\underline{w}$ is an eigenvector of J corresponding to the same eigenvalue. How can you multiply the eigenvectors of J by L to get the eigenvectors of K? [3]

   (c) Normalise all of the eigenvectors of J and K and hence create orthonormal matrices of eigenvectors P and Q, determine the singular value decomposition of L and use it to give L^+, the Moore-Penrose inverse of L. [2]

   (d) Verify that $L^+LL^+ = L^+$ for your matrices and use algebra to show this must always hold for any matrix and its Moore-Penrose inverse. [3]