Math2103 Linear Algebra: Assignment 5 (December 2015)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You can use Maple at any point and can email me any worksheets you created.

You are reminded that plagiarism is a serious offense and when it is detected you will be punished. Feel free to discuss the questions in general with myself and your colleagues but the work attempted must be yours alone. 2^{y-1} marks will be forfeited if you hand your work in y days after the deadline. Let the last 3 digits of your registration number be p, q, and r.

1. Let vector addition and scalar multiplication be defined as in Handout 1 for vectors $\underline{v} := \begin{pmatrix} a \\ b \end{pmatrix}$ and $\underline{w} := \begin{pmatrix} c \\ d \end{pmatrix}$ in $\mathbb{R}^2$ as follows.

$$\underline{v} + \underline{w} := \begin{pmatrix} a - qc \\ b - 2d + p \end{pmatrix} \quad , \quad \alpha \times \underline{v} := \begin{pmatrix} \alpha b \\ a \end{pmatrix}$$

   (a) Carefully evaluate the expressions on both sides of axiom A3 and hence classify exactly which vectors satisfy it. [3]

   (b) Verify axioms A4 and A5 hold for a $\underline{0}$ which isn't $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and identify $(-\underline{v})$. [2]

   (c) Show that there is exactly one vector $\underline{w}$ for which axiom S2 holds but find what values of p and q (not necessarily from your registration number) would make axiom S2 true always. [3]

   (d) Choose either axiom S3 or S4 and find all vectors and scalars for which the axioms are true and give a counterexample to show they are false in general. [3]

2. (a) Using Maple with your matrix from assignment 3, apply the Raleigh method to count how many iterations it takes to get to within 1% of each eigenvalue after starting from an integral valued vector not too far from the different eigenvectors, and explain why some of your eigenvalues converge quicker than others. [5]

   (b) Using algebra show that if λ is an eigenvalue of matrix A then λ^k is an eigenvalue of matrix A^k for any integer k and that $A - jI$ has eigenvalue $\lambda - j$ (and they all share the same eigenvector). Hence explain how to find a matrix M which has no eigenvalues of multiplicity greater than 1 but M^4 has just one eigenvalue μ. [4]