Math 2103 Handout 1: Vector Space Axioms

Given any set V of ordered lists of elements from a number system (called vectors in general) which can be added together and multiplied by a number from that same system (a scalar), the following ten axioms must hold for V to be a **vector space**.

- Additive Rules:

  A1. Closure: if $\underline{v}$ and $\underline{w}$ are in V then $(\underline{v} + \underline{w}) \in V$. ($\in$ means "is in")

  A2. Commutativity: if $\underline{v}$ and $\underline{w}$ are in V then $(\underline{v} + \underline{w}) = (\underline{w} + \underline{v})$.

  A3. Additive Associativity: if $\underline{u}$, $\underline{v}$ and $\underline{w}$ are in V then $(\underline{u} + \underline{v}) + \underline{w} = \underline{u} + (\underline{v} + \underline{w})$

  A4. Additive Identity: there exists a vector $\underline{0} \in V$ such that $(\underline{v} + \underline{0}) = \underline{v}$ for all $\underline{v} \in V$.

  A5. Inverse: For each $\underline{v} \in V$ there is an element $(-\underline{v}) \in V$ such that $\underline{v} + (-\underline{v}) = \underline{0}$.

- Scalar Multiplication Rules:

  S1. Closure: if $\underline{v} \in V$ then $\alpha \times \underline{v} \in V$ for all scalars α.

  S2. Scalar Distributivity: $\alpha \times (\underline{v} + \underline{w}) = (\alpha \times \underline{v}) + (\alpha \times \underline{w})$ for all scalars α and all vectors $\underline{v}$ and $\underline{w}$.

  S3. Vector Distributivity: $(\alpha + \beta) \times \underline{v} = (\alpha \times \underline{v}) + (\beta \times \underline{v})$ for all scalars α and β and all $\underline{v} \in V$.

  S4. Scalar Associativity: $\alpha \times (\beta \times \underline{v}) = (\alpha \times \beta) \times \underline{v}$ for all scalars α and β and all $\underline{v} \in V$.

  S5. Scalar Identity: $1 \times \underline{v} = \underline{v}$ for the scalar identity element 1 and any $\underline{v} \in V$.