Math 2103 (2016/17)
Assignment 1: Vector Spaces

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

1. You have selected a matrix M that I have created to have some special properties.
   (a) Using $\mathbb{Z}_{13}$ use either row operations or the adjoint method to find the inverse of M and check your answer is correct. [4]
   (b) Show that the product of this matrix and M is not the identity in $\mathbb{Z}_{11}$, but investigate and try to find a non-trivial such matrix, explaining your thought processes as you go along. [3]
   (c) In $\mathbb{Z}_{13}$ verify that $\begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$ is an eigenvector of M and find the other eigenvalues and their eigenspaces. Check that the eigenspaces are also the eigenspaces of the inverse of M and explain how the eigenvalues are related to those of M. [5]
2. What properties of the underlying number system are necessary for matrices built with them to have Right Scalar Associativity: $B(\alpha \times C) = \alpha \times (BC)$; use the standard definition $\alpha \times A := [\alpha a_{i,j}]$ and give all details of the algebra used. [4]
3. Create an isomorphism T between $\mathbb{R}^3$ and $\mathbb{P}^2$ such that $T(\underline{u}) = 2x^2 - 5x + 1$ where $\underline{u}$ is one of the other eigenvectors of M. Give $T(\begin{pmatrix} e \\ f \\ g \end{pmatrix})$ in general as your answer. [4]

$$M_1 := \begin{pmatrix} 2 & 7 & 2 \\ 3 & 0 & 10 \\ 5 & 2 & 12 \end{pmatrix}$$

$$M_2 := \begin{pmatrix} 8 & 12 & 1 \\ 11 & 11 & 4 \\ 7 & 1 & 1 \end{pmatrix}$$

$$M_3 := \begin{pmatrix} 11 & 10 & 8 \\ 1 & 11 & 3 \\ 5 & 9 & 6 \end{pmatrix}$$

$$M_4 := \begin{pmatrix} 7 & 8 & 9 \\ 3 & 12 & 9 \\ 2 & 1 & 10 \end{pmatrix}$$

$$M_5 := \begin{pmatrix} 1 & 9 & 12 \\ 2 & 7 & 7 \\ 3 & 3 & 9 \end{pmatrix}$$

$$M_6 := \begin{pmatrix} 0 & 4 & 10 \\ 10 & 3 & 7 \\ 4 & 11 & 3 \end{pmatrix}$$