Math 2103 (2016/17)
Assignment 2: Linear Transformations

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

The numbers represented by a, b and c should be replaced by the last three digits of your registration number in that order. For instance, if my registration number was 20015374 then I would take $a = 3$, $b = 7$ and $c = 4$.

1. Let S be the linear transformation such that

$$S(e + fi) = (ea + fc) \times x + (4e + 5f)$$

   You can work in either $\mathbb{Z}_{13}$ or the real numbers, whichever you prefer.

   (a) Show that S is an isomorphism. [2]

   (b) Find the inverse transformation S^{-1} and verify by substitution that when combined with S (on either left or right) it gives an identity transformation. [4]

   (c) Determine the underlying matrix M of S and find M^{-1}. [1]

   (d) Create a linear transformation R from $\mathbb{P}_1$ to $\mathbb{C}$ with a kernel of dimension 1 and determine $R \circ S$ and $S \circ R$ and the dimensions of their kernels. Investigate if there can exist a pair of linear transformations between vector spaces of dimension 2 each with a 1 dimensional kernel such that their composition has a kernel which is not of dimension 1. [5]

2. Let T be a transformation from polynomials with real coefficients which have maximum degree n to a column vector of its evaluations such that:

$$T(p(x)) = \begin{pmatrix} p(b) \\ p(2) \\ p(-1) \end{pmatrix}$$

   (a) Verify the two linear transformation axioms for T. [3]

   (b) Determine the kernel and image of T (in general, but making sure to account for special cases when n is small) and check that their dimensions match the formula given in class. [5]