Math 2103 (2016/17)
Assignment 3: Linear Operators

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

The numbers represented by a and b should be replaced by the last two *non-zero* digits of your registration number in that order. For instance, if my registration number was 20015374 then I would take $a = 7$ and $b = 4$.

1. (a) Let T be the linear operator that you chose, find the kernel and image of T and their dimensions. [2]

   (b) Identify the underlying matrix of T, calculate its eigenvalues and eigenvectors and hence determine all T-invariant subspaces of dimension 2. [5]

   (c) Find all of the T-invariant subspaces not already mentioned, explaining why they are different from those already found and why you haven't missed any. [4]

2. (a) Create a 2×2 real valued matrix M with no zeros and no real eigenvalues which is different to the matrices created by everyone else in the class. Full marks will be given for ensuring M has $(a+bi)$ as an eigenvalue. Find the complex eigenvectors. [4]

   (b) Create a 4×4 matrix $F := \begin{bmatrix} M & O \\ O & M \end{bmatrix}$ where O is the all zeros matrix and use this matrix to create a linear operator S which acts on the space of 2×2 matrices. Find standard bases for the two different S-invariant subspaces of dimension 2 and show that they are different spaces. Is the sum of these two spaces equal to $\mathbb{M}_{2,2}$? [5]

$$T_1(ax^2+bx+c) := (32\,a+40\,b-40\,c)\,x^2+(-41\,a-52\,b+55\,c)\,x-17\,a-22\,b+25\,c$$

$$T_2(ax^2+bx+c) := (-20\,a-23\,b-50\,c)\,x^2+(4\,a+7\,b+10\,c)\,x+8\,a+8\,b+20\,c$$

$$T_3(ax^2+bx+c) := (-13\,a-20\,b-4\,c)\,x^2+(-3\,a-4\,b)\,x+78\,a+116\,b+18\,c$$

$$T_4(ax^2+bx+c) := (58\,a-102\,b+11\,c)\,x^2+(15\,a-27\,b+3\,c)\,x-160\,a+276\,b-29\,c$$

$$T_5(ax^2+bx+c) := (-5\,a+21\,b-24\,c)\,x^2+(-6\,a+22\,b-24\,c)\,x-3\,a+11\,b-12\,c$$

$$T_6(ax^2+bx+c) := (15\,a-18\,b+24\,c)\,x^2+(20\,a-16\,b+24\,c)\,x+5\,a+2\,c$$