## Math 2103 (2017/18) <br> Assignment 1: Vector Spaces

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. Maple worksheets can be submitted showing the working done.

The numbers represented by $a, b$ and $c$ should be replaced by the last three digits of your registration number in that order, and if the digit is 0 it is replaced by -3 . For instance, if my registration number was 20015270 then i would take $a=2, b=7$ and $c=-3$.

1. You each have the following matrix $M$ from $Z t$ to deal with.

Shannon: $M:=\left(\begin{array}{cccc}1 & 4 & 4 & 12 \\ 7 & 10 & 11 & 5 \\ 8 & 5 & 3 & 0 \\ 4 & 0 & 9 & 9\end{array}\right)$, Illya: $M:=\left(\begin{array}{cccc}11 & 4 & 4 & 12 \\ 8 & 8 & 3 & 6 \\ 2 & 12 & 9 & 7 \\ 10 & 6 & 0 & 12\end{array}\right)$
(a) Show that the determinant of your $M$ is 0 using row operations and cofactor expansions, remembering to replace any large numbers formed by their equivalent numbers in $\mathbb{Z}_{13}$. [4]
(b) Determine the eigenvalues of $M$ in $\mathbb{Z}_{13}$ by finding the determinant of $M-\lambda I$ and then factor it by substituting possible values into the polynomial.
(c) Find a spanning space for the eigenvectors of $M$ belonging to the eigenvalue of multiplicity 2 with as many zeroes as possible in the basis vectors.
2. (a) Using your registration number, find a vector $\underline{w}$ in $\mathbb{P}_{2}$ which is in the space spanned by $\underline{v}_{1}:=3 x^{2}+a x-4$ and $\underline{v}_{2}:=b x^{2}+x+1$ which has a coefficient of 0 for its $x^{2}$ term. [2]
(b) Now create a basis set for $\mathbb{R}^{3}$ which includes $\underline{u}_{1}:=\left(\begin{array}{c}-2 \\ c \\ 5\end{array}\right)$ and explain why it is a basis.
(c) Hence make a linear transformation $T$ such that $T\left(\underline{u}_{1}\right)=\underline{v}_{1}, T\left(\underline{u}_{2}\right)=\underline{v}_{2}$ and $T\left(\underline{u}_{3}\right)=\underline{w}$. Check your transformation does give the expected results once you have it in its final form.
(d) What space is the kernel of $T$ ? Why must it have dimension 1 no matter what choices you made in part (c)?

