## Math 2103 (2017/18)

Assignment 2: Linear Transformations and Operators

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. Maple worksheets can be submitted showing the working done.

The numbers represented by $p$ and $q$ should be replaced by the last two non-zero digits of your registration number in that order. For instance, if my registration number was 20015270 then i would take $p=2$ and $q=7$.

1. For this question the linear transformations $T$ are as randomly selected:

$$
\begin{aligned}
& T_{1}\left(\left(\begin{array}{ll}
a & b \\
c & d
\end{array}\right)\right)=x^{2}(2 a+3 b+3 c+4 d)+x(a+4 b+3 c+2 d)+(a+4 b+3 c+2 d) \\
& T_{2}\left(\left(\begin{array}{ll}
a & b \\
c & d
\end{array}\right)\right)=x^{2}(a+2 b+2 c+3 d)+x(a+5 b+2 c+5 d)+(2 a+4 b+4 c+6 d)
\end{aligned}
$$

(a) Verify that your $T$ satisfies the two linear transformation axioms.
(b) Determine the kernel and image of your $T$ and carefully produce a basis set for both, making sure that your dimensions add up and your bases are independent.
(c) If $S$ is a linear transformation from $\mathbb{P}_{2}$ to $\mathbb{M}_{2,2}$, what possible combinations of dimensions can be achieved for the image spaces of $T \circ S$ and $S \circ T$ ? For each claimed possibility give a simple example of an $S$ which demonstrates this.
2. (a) Using your knowledge of diagonalisation or otherwise, create a matrix with no zeroes in that has $p$ and $q$ as its eigenvalues.
(b) Convert your matrix into a linear operator $R$ in $\mathbb{C}$ and find all $R$-invariant subspaces. Evaluate $R \circ R$ and verify that the same subspaces are still invariant under the linear operator $R \circ R$.
(c) Now define $S$ to be the linear operator such that $S(e+f i)=(|q-p| f)-(q e) i$. Explain why there are no $S$-invariant subspaces of dimension 1 , but find all ones of that dimension for $S \circ S$.
(d) Predict what you believe will happen with regards invariant subspaces for $R \circ S$ and then check it via the underlying matrices.

