## Math 2103 (2017/18)

## Assignment 3: Inner products and orthogonality

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them.

1. This is your quadratic form for this question:

$$
f(w, x, y, z):=4 w^{2}+2 w z+6 x^{2}-4 x y+6 x z+8 y^{2}+5 z^{2}
$$

(a) Which symmetric matrix $M$ satisfies $f(w, x, y, z)=\left(\begin{array}{llll}w & x & y & z\end{array}\right) M\left(\begin{array}{l}w \\ x \\ y \\ z\end{array}\right)$ ?
(b) Use Cholesky to factorise $M$ as $U^{T} U$ for an upper triangular matrix $U$.
(c) Express $f(w, x, y, z)$ as a sum of squares with positive rational coefficients.
(d) Calculate the characteristic polynomial of $M$ and use your Maple/calculus skills to show all roots of it are greater than 1 and explain the significance of this.
2. Let $A:=\left(\begin{array}{cccc}2 & 3 & -1 & 3 \\ 1 & 3 & 2 & 3\end{array}\right)$.
(a) Find the eigenvalues and eigenvectors of $A A^{T}$ and the eigenvectors corresponding to the eigenvalue 0 of $A^{T} A$.
(b) Use these eigenvectors to carefully create the orthogonal matrices $P$ and $Q$ such that $A=P E Q^{T}$ and check that all the necessary properties hold.
3. Let $H$ be a complex matrix such that $\bar{H}=H^{T}$. Use the complex inner product to prove (in the same way as we did for symmetric matrices) that the eigenvalues of $H$ are real numbers, giving the properties used at each stage.
4. Using the fact that any vector in a space $S$ can be written as $\underline{v}+\underline{w}$ where $\underline{v} \in W^{\perp}:=\{\underline{u} \in$ $S ;<\underline{u}, \underline{w}>=0, \forall \underline{w} \in W\}$ and $\underline{w} \in W$, explain why $W \subseteq\left(W^{\perp}\right)^{\perp}$.

